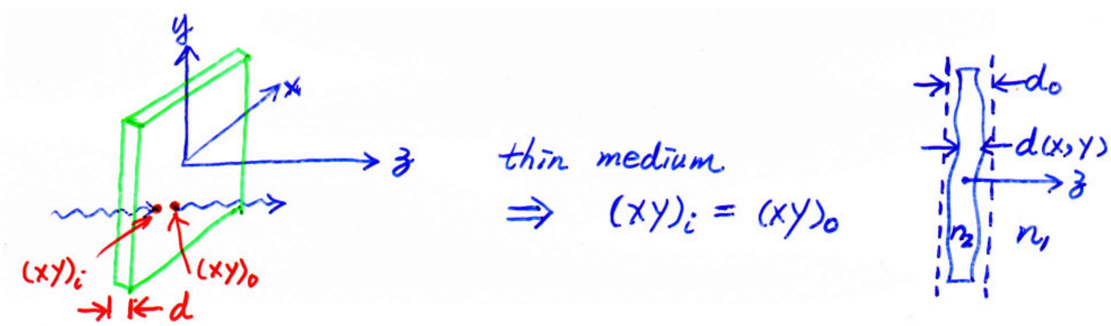
Chapter 8 Effects of Partial Coherence on Image Formation Process: A Rigorous Statistical Analysis

In this chapter, a rigorous mathematical framework will be presented to illustrate the impact of partial coherence of light on image formation. The concepts of statistical optics introduced in the previous chapters will be used to facilitate the discussion.

8.1 Preliminary Considerations

(a) Effects of a thin transparent object on mutual coherence function

First let us consider a thin transparent slab as shown below



The time-delay suffered by the transmitted wave $u_o(x,y;t) = B(x,y)u_i(x,y;t-\delta)$ is given by

$$\begin{aligned}\delta &= \frac{d(x,y)}{v_2} + \frac{d_0 - d(x,y)}{v_1} \\ &= \frac{[n_2(x,y) - n_1]}{c} d(x,y) + \frac{n_1 d_0}{c} \\ &= \frac{\Delta n(x,y)}{c} d(x,y) + \frac{n_1 d_0}{c}\end{aligned}.$$

By invoking $u_i(P;t) = A_i(P,t)\cdot e^{-j2\pi\bar{\nu}t}$ of narrowband light, the mutual coherence function behind the thin transparent slab can be calculated

$$\begin{aligned}\Gamma_o(P_1,P_2;\tau) &= \langle u_o(P_1;t+\tau)u_o^*(P_2;t)\rangle \\ &= B(P_1)B(P_2)\langle u_i(P_1;t+\tau-\delta(P_1))u_i^*(P_2;t-\delta(P_2))\rangle \\ &= B(P_1)B(P_2)e^{j2\pi\bar{\nu}\delta(P_1)-j2\pi\bar{\nu}\delta(P_2)}\cdot\langle A_i(P_1;t+\tau-\delta(P_1)+\delta(P_2))A_i^*(P_2;t)\rangle e^{-j2\pi\bar{\nu}\tau}\cdot \\ &= t(P_1)t^*(P_2)\Gamma_i(P_1,P_2;\tau)\end{aligned}$$

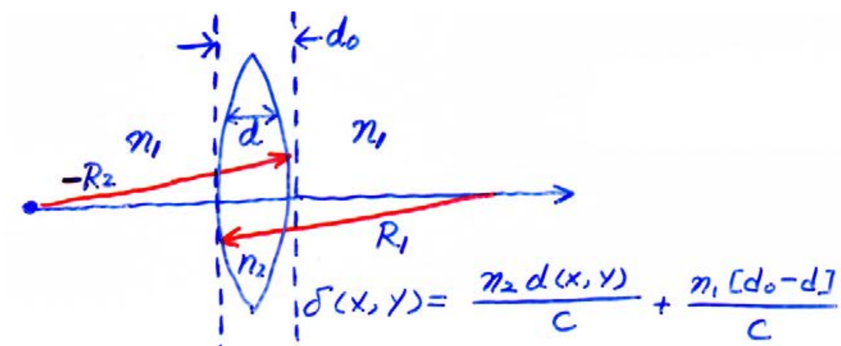
As the quasi-monochromatic condition $|\delta(P_1)-\delta(P_2)| << \tau_c = 1/\Delta\nu$ is fulfilled, $\langle A_i(P_1;t+\tau-\delta(P_1)+\delta(P_2))A_i^*(P_2;t)\rangle e^{-j2\pi\bar{\nu}\tau}$ will be independent of $\delta(P_1)$ and $\delta(P_2)$.

If $\tau << \tau_c$, we obtain

$$J_o(P_1,P_2) = t(P_1)t^*(P_2)J_i(P_1,P_2) \qquad \text{(A).}$$

(b) Time delays induced by a thin lens

Refer to the following figure



,

where

$$\begin{aligned}d(x,y) &= d_0 - R_1\Big(1-\sqrt{1-\frac{x^2+y^2}{R_1^2}}\Big) + R_2\Big(1-\sqrt{1-\frac{x^2+y^2}{R_2^2}}\Big) \\ &\approx d_0 - \frac{x^2+y^2}{2}\Big(\frac{1}{R_1}-\frac{1}{R_2}\Big)\end{aligned}.$$

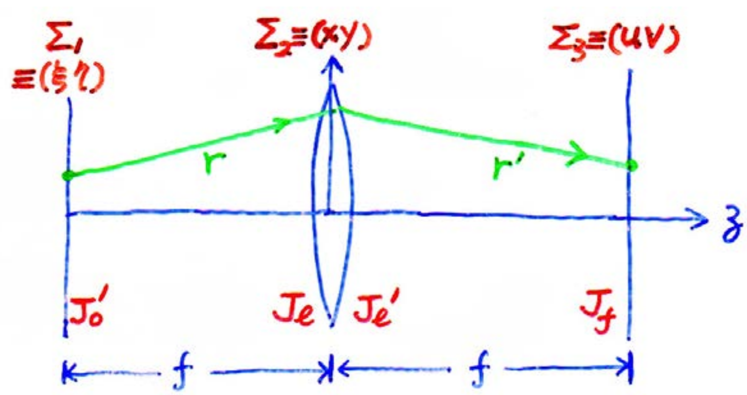
Therefore,

$$
\begin{aligned}
\delta(x,y) &= \frac{n_2 d_0}{c} - \frac{1}{2}\frac{(n_2 - n_1)}{c}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)(x^2 + y^2) \\
&= \frac{n_2 d_0}{c} - \frac{1}{2c\,f}(x^2 + y^2) \\
&= \frac{n_2 d_0}{c} - \frac{1}{2\bar{\lambda}\bar{\nu}\,f}(x^2 + y^2)
\end{aligned}
$$

Neglecting the constant phase term of $\frac{n_2 d_0}{c}$, then we can derive

$$
t_l(x,y) = e^{j2\pi\bar{\nu}\,\delta(x,y)} = e^{-j\frac{\pi}{\bar{\lambda} f}(x^2+y^2)}, \text{ when } \left|\delta(P_1) - \delta(P_2)\right| << \tau_c . \qquad \text{(B)}
$$

(c) Focal-plane-to-focal-plane coherence relationships



On the front focal plane Σ_1, the mutual intensity function (MIF) of an quasi-monochromatic light reads $J_0'(\vec{\xi}_1;\vec{\xi}_2) \equiv J_0'(\xi_1,\eta_1;\xi_2,\eta_2)$, where the prime denotes the light field leaves the front focal plane. After propagating a distance of *f*, it will arrive at the lens. We can calculate the corresponding mutual intensity with

$$
\begin{aligned}
J_l(\vec{x}_1;\vec{x}_2) &\equiv J_l(x_1,y_1;x_2,y_2) \\
&= \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2}\, J_0{}'(\vec{\xi}_1;\vec{\xi}_2) e^{-j\frac{2\pi}{\bar{\lambda}}(r_2-r_1)} \frac{\chi(\theta_1)}{\bar{\lambda} r_1} \frac{\chi(\theta_2)}{\bar{\lambda} r_2} .
\end{aligned}
$$

By taking $\chi(\theta_1) \approx \chi(\theta_1) \sim 1$, $\frac{1}{\bar{\lambda} r_1} \cong \frac{1}{\bar{\lambda} r_2} \cong \frac{1}{\bar{\lambda} f}$, and

$r_2 - r_1 = \left|\vec{R}_2 - \vec{\xi}_2\right| - \left|\vec{R}_1 - \vec{\xi}_1\right| \cong \frac{1}{2f}\left[\left|\vec{x}_2 - \vec{\xi}_2\right|^2 - \left|\vec{x}_1 - \vec{\xi}_1\right|^2\right]$, $J_l(\vec{x}_1;\vec{x}_2)$ can be approximated as

$$
\begin{aligned}
J_l(\vec{x}_1;\vec{x}_2) &\equiv J_l(x_1,y_1;x_2,y_2) \\
&\cong \frac{1}{(\bar{\lambda} f)^2} e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{x}_2|^2 - |\vec{x}_1|^2)} \cdot \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2}\, J_0{}'(\vec{\xi}_1;\vec{\xi}_2) e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{\xi}_2|^2 - |\vec{\xi}_1|^2)} \cdot e^{j\frac{2\pi}{\bar{\lambda} f}(\vec{\xi}_2\cdot\vec{x}_2 - \vec{\xi}_1\cdot\vec{x}_1)} .
\end{aligned}
$$

From equation (A) and (B), the mutual intensity after passing through the thin lens becomes

$$
\begin{aligned}
J_l{}'(\vec{x}_1;\vec{x}_2) &= J_l(\vec{x}_1;\vec{x}_2) \cdot e^{j\frac{\pi}{\bar{\lambda} f}(|\vec{x}_2|^2 - |\vec{x}_1|^2)} \\
&= \frac{1}{(\bar{\lambda} f)^2} \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2}\, J_0{}'(\vec{\xi}_1;\vec{\xi}_2) e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{\xi}_2|^2 - |\vec{\xi}_1|^2)} \cdot e^{j\frac{2\pi}{\bar{\lambda} f}(\vec{\xi}_2\cdot\vec{x}_2 - \vec{\xi}_1\cdot\vec{x}_1)} .
\end{aligned}
$$

The mutual intensity on the rear focal plane is derived as

$$
\begin{aligned}
J_f(\vec{u}_1;\vec{u}_2) &= \frac{1}{(\bar{\lambda} f)^2} e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{u}_2|^2 - |\vec{u}_1|^2)} \cdot \int_{\Sigma_2} dA_{\vec{x}_1} \int_{\Sigma_2} dA_{\vec{x}_2}\; J_l{}'(\vec{x}_1;\vec{x}_2) \cdot e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{x}_2|^2 - |\vec{x}_1|^2)} e^{j\frac{2\pi}{\bar{\lambda} f}(\vec{u}_2\cdot\vec{x}_2 - \vec{u}_1\cdot\vec{x}_1)} \\
&= \frac{1}{(\bar{\lambda} f)^2} e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{u}_2|^2 - |\vec{u}_1|^2)} \cdot \int_{\Sigma_2} dA_{\vec{x}_1} \int_{\Sigma_2} dA_{\vec{x}_2}\; \frac{1}{(\bar{\lambda} f)^2} e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{x}_2|^2 - |\vec{x}_1|^2)} \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2}\, J_0{}'(\vec{\xi}_1;\vec{\xi}_2) e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{\xi}_2|^2 - |\vec{\xi}_1|^2)} \cdot e^{j\frac{2\pi}{\bar{\lambda} f}(\vec{\xi}_2\cdot\vec{x}_2 - \vec{\xi}_1\cdot\vec{x}_1 + \vec{u}_2\cdot\vec{x}_2 - \vec{u}_1\cdot\vec{x}_1)} . \\
&= \frac{1}{(\bar{\lambda} f)^4} \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2}\; J_0{}'(\vec{\xi}_1;\vec{\xi}_2) \cdot e^{-j\frac{\pi}{\bar{\lambda} f}(|\vec{u}_2|^2 - |\vec{u}_1|^2 + |\vec{\xi}_2|^2 - |\vec{\xi}_1|^2)} \cdot \mathscr{G}(\vec{\xi}_1;\vec{\xi}_2)
\end{aligned}
$$

under paraxial conditions. Here

$$\mathscr{G}(\vec{\xi}_1;\vec{\xi}_2)=\int_{\Sigma_2}dA_{\vec{x}_1}\int_{\Sigma_2}dA_{\vec{x}_2}\; e^{-j\frac{\pi}{\bar{\lambda}f}(|\vec{x}_2|^2-|\vec{x}_1|^2)}\cdot e^{j\frac{2\pi}{\bar{\lambda}f}(\vec{\xi}_2+\vec{u}_2)\cdot\vec{x}_2-j\frac{2\pi}{\bar{\lambda}f}(\vec{\xi}_1+\vec{u}_1)\cdot\vec{x}_1}$$
$$=(\bar{\lambda}f)^2 e^{j\frac{\pi}{\bar{\lambda}f}(|\vec{\xi}_2+\vec{u}_2|^2-|\vec{\xi}_1+\vec{u}_1|^2)}.$$

Finally, we obtain

$$J_f(\vec{u}_1;\vec{u}_2)=\frac{1}{(\bar{\lambda}f)^2}\int_{\Sigma_1}dS_{\vec{\xi}_1}\int_{\Sigma_1}dS_{\vec{\xi}_2}\; J_0'(\vec{\xi}_1;\vec{\xi}_2)\cdot e^{j\frac{2\pi}{\bar{\lambda}f}(\vec{\xi}_2\cdot\vec{u}_2-\vec{\xi}_1\cdot\vec{u}_1)},$$

which implies the mutual intensities in the front and back focal planes of a thin positive lens form a 4D Fourier transform pair. This relation serves as the foundation of Fourier optics with partially coherent light.

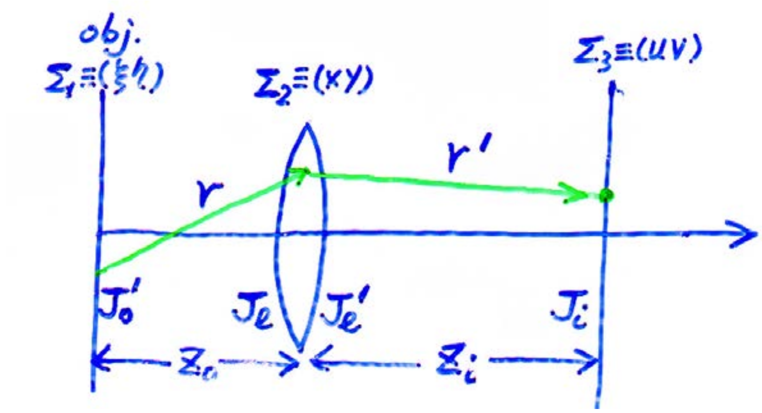
Let $\vec{u}_1=\vec{u}_2=\vec{u}$, $I_f(\vec{u})=J_f(\vec{u};\vec{u})=\frac{1}{(\bar{\lambda}f)^2}\int_{\Sigma_1}dS_{\vec{\xi}_1}\int_{\Sigma_1}dS_{\vec{\xi}_2}\; J_0'(\vec{\xi}_1;\vec{\xi}_2)\cdot e^{j\frac{2\pi}{\bar{\lambda}f}\vec{u}\cdot(\vec{\xi}_2-\vec{\xi}_1)}$.

Note that these equations are valid as long as quasi-monochromatic conditions hold, *i.e.*,

$$\tau_2-\tau_1=(\frac{r_2+r_2'}{c}+\delta_2)-(\frac{r_1+r_1'}{c}+\delta_1)<<\tau_c \quad\Rightarrow\left|\frac{\vec{\xi}_2\cdot\vec{u}_2-\vec{\xi}_1\cdot\vec{u}_1}{fc}\right|<<\tau_c \quad\Rightarrow\quad \frac{L_0L_f}{f}<<l_c.$$

By use of $L_0=L_f=5cm,\ f=1m$, we find $l_c(\textit{coherent length of light})>>2.5\,mm$.

(d) Object-Image Coherence Relations for a Thin Lens



Assuming the amplitude transmittance function of the lens can be expressed as

$$t_l(\vec{x}) \equiv t_l(x,y) = P(\vec{x})e^{-j\frac{\pi}{\bar{\lambda} f}|\vec{x}|^2},$$

the mutual intensity immediately behind the lens shall be read as

$$J_l'(\vec{x}_1,\vec{x}_2) = \frac{1}{(\bar{\lambda} z_o)^2} P(\vec{x}_1)P^*(\vec{x}_2)e^{-j\frac{\pi}{\bar{\lambda}}(\frac{1}{z_o}-\frac{1}{f})(|\vec{x}_2|^2-|\vec{x}_1|^2)} \cdot \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2} J_o'(\vec{\xi}_1,\vec{\xi}_2) e^{-j\frac{\pi}{\bar{\lambda} z_o}(|\vec{\xi}_2|^2-|\vec{\xi}_1|^2)} \cdot e^{j\frac{2\pi}{\bar{\lambda} z_o}(\vec{\xi}_2\cdot\vec{x}_2-\vec{\xi}_1\cdot\vec{x}_1)}.$$

The mutual intensity in the image plane becomes

$$J_i(\vec{u}_1;\vec{u}_2) = \frac{1}{(\bar{\lambda} z_o)^2(\bar{\lambda} z_i)^2}\cdot e^{-j\frac{\pi}{\bar{\lambda} z_i}(|\vec{u}_2|^2-|\vec{u}_1|^2)} \cdot \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2} J_o'(\vec{\xi}_1;\vec{\xi}_2) e^{-j\frac{\pi}{\bar{\lambda} z_o}(|\vec{\xi}_2|^2-|\vec{\xi}_1|^2)} \cdot$$
$$\int_{\Sigma_2} dA_{\vec{x}_1} \int_{\Sigma_2} dA_{\vec{x}_2} P(\vec{x}_1)P^*(\vec{x}_2) e^{-j\frac{\pi}{\bar{\lambda}}(\frac{1}{z_o}+\frac{1}{z_i}-\frac{1}{f})(|\vec{x}_2|^2-|\vec{x}_1|^2)} e^{j\frac{2\pi}{\bar{\lambda}}[(\frac{\vec{u}_2}{z_i}+\frac{\vec{u}_1}{z_o})\cdot\vec{x}_2-(\frac{\vec{u}_1}{z_i}+\frac{\vec{u}_2}{z_o})\cdot\vec{x}_1]},$$

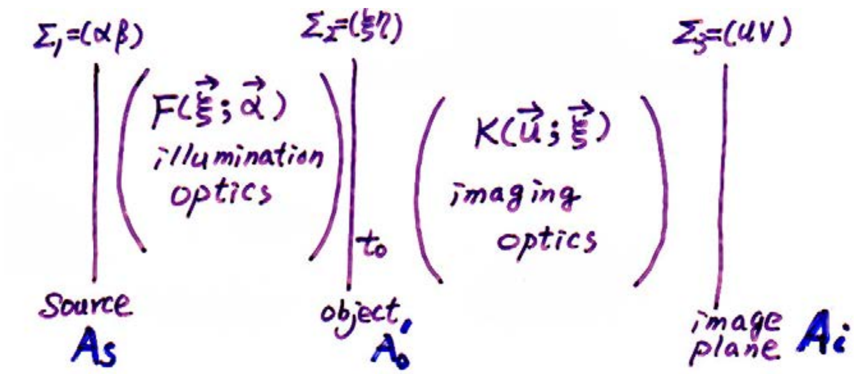
$$\boxed{J_i(\vec{u}_1;\vec{u}_2) = \int_{\Sigma_1} dS_{\vec{\xi}_1} \int_{\Sigma_1} dS_{\vec{\xi}_2} J_o'(\vec{\xi}_1;\vec{\xi}_2) K(\vec{u}_1;\vec{\xi}_1)\cdot K^*(\vec{u}_2;\vec{\xi}_2)},$$

where $K(\vec{u};\vec{\xi}) = \frac{e^{j\frac{\pi}{\bar{\lambda}}(\frac{|\vec{u}|^2}{z_i}+\frac{|\vec{\xi}|^2}{z_o})}}{(\bar{\lambda} z_o)(\bar{\lambda} z_i)}\cdot\int_{\Sigma_2} dA\, P(\vec{x}) e^{-j\frac{2\pi}{\bar{\lambda}}(\frac{\vec{u}}{z_i}+\frac{\vec{\xi}}{z_o})\cdot\vec{x}}$ is the Green function, which maps a point source at $\vec{\xi}$ on Σ_1 to a position $\vec{u}$ in the image plane.

8.2 Methods for Calculating Image Intensity

In general, two methods can be exploited to deduce an image distribution using a partially coherent light source.

(a) Integration over the source



I) The time-varying phasor amplitude to the right of the object can be expressed as $A_o'(\vec{\xi},\vec{\alpha};t)=t_o(\vec{\xi})F(\vec{\xi},\vec{\alpha})A_s(\vec{\alpha};t-\delta)$.

The phasor amplitude from source point ($\alpha\beta$) and reaching the image plane (u, v) is given by

$$A_i(\vec{u},\vec{\alpha};t)=\int_{\Sigma_2} K(\vec{u},\vec{\xi})t_o(\vec{\xi})F(\vec{\xi},\vec{\alpha})A_s(\vec{\alpha};t-\delta_1-\delta_2)dA_{\vec{\xi}}\ .$$

The image intensity can be expressed as

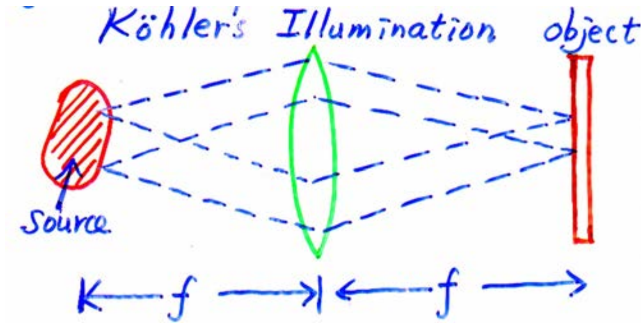
$$\begin{aligned} I_i(\vec{u},\vec{\alpha}) &= <\left|A_i(\vec{u},\vec{\alpha};t)\right|^2> \\ &= \int_{\Sigma_2} dA_{\vec{\xi}_1}\int_{\Sigma_2} dA_{\vec{\xi}_2} K(\vec{u},\vec{\xi}_1)t_o(\vec{\xi}_1)F(\vec{\xi}_1,\vec{\alpha})\cdot K^*(\vec{u},\vec{\xi}_2)t_o^*(\vec{\xi}_2)F^*(\vec{\xi}_2,\vec{\alpha})<A_s(\vec{\alpha};t-\delta_1-\delta_2)A_s^*(\vec{\alpha};t-\delta_1'-\delta_2')>\end{aligned}$$

If $\left|\delta_1+\delta_2-\delta_1'-\delta_2'\right|<<\tau_c$, then $<A_s(\vec{\alpha};t-\delta_1-\delta_2)A_s^*(\vec{\alpha};t-\delta_1'-\delta_2')>=I_s(\vec{\alpha})$.

Thus, $I_i(\vec{u})$ becomes

$$I_i(\vec{u})=\int_{\Sigma_1} I_s(\vec{\alpha})\,dS_{\vec{\alpha}}\int_{\Sigma_2} dA_{\vec{\xi}_1}\int_{\Sigma_2} dA_{\vec{\xi}_2} K(\vec{u};\vec{\xi}_1)K^*(\vec{u};\vec{\xi}_2)\cdot F(\vec{\xi}_1;\vec{\alpha})F^*(\vec{\xi}_2;\vec{\alpha})\cdot t_o(\vec{\xi}_1)t_o^*(\vec{\xi}_2)\,.$$

With a Kohler's illumination optics, the object is illuminated by a source effectively at infinite distance as shown below



$$\Rightarrow \quad F(\vec{\xi}, \vec{\alpha}) = \frac{1}{\bar{\lambda} f} e^{-j\frac{2\pi}{\bar{\lambda} f}(\vec{\xi}\cdot\vec{\alpha})} .$$

In this illumination condition,

$$I_i(\vec{u}) = \int_{\Sigma_1} I_s(\vec{\alpha})\, dS_\alpha \int_{\Sigma_2} dA_{\xi_1} \int_{\Sigma_2} dA_{\xi_2} K(\vec{u}; \vec{\xi}_1) K^*(\vec{u}; \vec{\xi}_2) \cdot \frac{1}{(\bar{\lambda} f)^2} e^{j\frac{2\pi}{\bar{\lambda} f}(\vec{\xi}_2 - \vec{\xi}_1)\cdot\vec{\alpha}} \cdot t_o(\vec{\xi}_1) t_o^*(\vec{\xi}_2) .$$

II) Representation of the source with an incident mutual intensity function

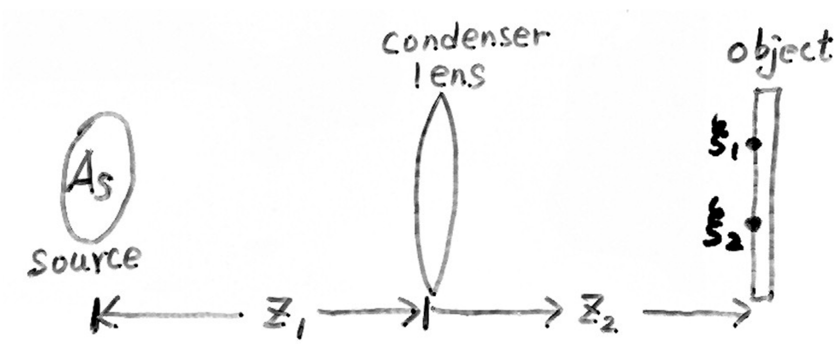
Note that under the quasi-monochromatic assumption, the image amplitude can be expressed as $A_i(\vec{u}, t) = \int_{\Sigma_2} K(\vec{u}, \vec{\xi}) t_o(\vec{\xi}) A_o(\vec{\xi}; t - \delta) dA_\xi$. The image intensity becomes

$$\begin{aligned} I_i(\vec{u}) &= < |A_i(\vec{u}, t)|^2 > \\ &= \int_{\Sigma_2} dA_{\vec{\xi}_1} \int_{\Sigma_2} dA_{\vec{\xi}_2} K(\vec{u}, \vec{\xi}_1) K^*(\vec{u}, \vec{\xi}_2) t_o(\vec{\xi}_1) t_o^*(\vec{\xi}_2) \cdot < A_o(\vec{\xi}_1; t - \delta_1) A_o^*(\vec{\xi}_2; t - \delta_2) > \end{aligned}$$

When $|\delta_1 - \delta_2| << \tau_c$, $< A_o(\vec{\xi}_1; t - \delta_1) A_o^*(\vec{\xi}_2; t - \delta_2) > = J_o(\vec{\xi}_1, \vec{\xi}_2)$.

$$I_i(\vec{u}) = \int_{\Sigma_2} dA_{\vec{\xi}_2} \int_{\Sigma_2} dA_{\vec{\xi}_1} K(\vec{u}, \vec{\xi}_1) K^*(\vec{u}, \vec{\xi}_2) \cdot t_o(\vec{\xi}_1) t_o^*(\vec{\xi}_2) \cdot J_o(\vec{\xi}_1, \vec{\xi}_2) .$$

Thus, to solve the imaging problem, $J_o(\vec{\xi}_1, \vec{\xi}_2)$ must be determined first.



By referring to the above figure, the coherence area of the source on the lens is about $A_c = (\bar{\lambda} z_1)^2 / A_s$. If A_s is sufficiently large, then A_c is much smaller than the area of the lens (A_l), i.e., $A_c = (\bar{\lambda} z_1)^2 / A_s << A_l$, so $A_l A_s >> (\bar{\lambda} z_1)^2$. In this case, we can view the lens as an effective source of incoherent illumination.

The MIF just before the lens can be derived from Van Citter-Zernike Theorem as

$$J_l(\vec{x}_1;\vec{x}_2) = \frac{e^{-j\frac{\pi}{\bar{\lambda} z_1}(|\vec{x}_2|^2 - |\vec{x}_1|^2)}}{(\bar{\lambda} z_1)^2} \cdot \int_{\Sigma_1} dS_{\vec{\alpha}}\ I_s(\vec{\alpha}) e^{j\frac{2\pi}{\bar{\lambda} z_1}\Delta\vec{x}\cdot\vec{\alpha}}, \quad where\ \Delta\vec{x} = \vec{x}_2 - \vec{x}_1 .$$

From the equation, we can further obtain

$$\begin{aligned} J_l'(\vec{x}_1;\vec{x}_2) &= P_c(\vec{x}_1) P_c^*(\vec{x}_2) \cdot e^{j\frac{\pi}{\bar{\lambda} f}(|\vec{x}_2|^2 - |\vec{x}_1|^2)} \cdot J_l(\vec{x}_1;\vec{x}_2) \\ &= \frac{1}{(\bar{\lambda} z_1)^2} |P_c(\vec{x}_1)|^2 \cdot \tilde{I}_s(\frac{\Delta\vec{x}}{\bar{\lambda} z_1}), \quad when\ |\Delta\vec{x}| \cong 0 \end{aligned},$$

where $\tilde{I}_s(\frac{\Delta\vec{x}}{\bar{\lambda} z_1})$ denotes the 2D FT of the source intensity distribution $I_s(\vec{\alpha})$ and is extremely narrow on $\Delta\vec{x}$ since $A_l A_s >> (\bar{\lambda} z_1)^2$. For practical purposes, $J_l'(\vec{x}_1;\vec{x}_2)$ can be viewed as a new source that is spatially incoherent with intensity distribution proportionally to $|P_c(\vec{x}_1)|^2$.

Invoking Van Cittert-Zernike theorem again, the MIF on the object plane becomes

$$\boxed{\begin{aligned} J_o(\vec{\xi}_1;\vec{\xi}_2) &= \frac{e^{-j\frac{\pi}{\bar{\lambda} z_2}(|\vec{\xi}_2|^2-|\vec{\xi}_1|^2)}}{(\bar{\lambda} z_2)^2}\cdot\int_{\Sigma_2}\int_{\Sigma_2} dA_{\vec{x}_2}\, dA_{\vec{x}_1}\, |P_c(\vec{x}_1)|^2\, \tilde{I}_s \delta(x_2-x_1) e^{j\frac{2\pi}{\bar{\lambda} z_2}(\vec{\xi}_2\cdot\vec{x}_2-\vec{\xi}_1\cdot\vec{x}_1)} \\ &= \frac{e^{-j\frac{\pi}{\bar{\lambda} z_2}(|\vec{\xi}_2|^2-|\vec{\xi}_1|^2)}}{(\bar{\lambda} z_2)^2}\cdot\int_{\Sigma_2} dA_{\vec{x}_1}\, \tilde{I}_s\cdot|P_c(\vec{x}_1)|^2\cdot e^{j\frac{2\pi}{\bar{\lambda} z_2}\Delta\vec{\xi}\cdot\vec{x}_1} \end{aligned}},$$

where $\Delta\vec{\xi}=\vec{\xi}_2-\vec{\xi}_1$, is independent of z_1 and any aberration that may exist in the illumination system. If $z_1=z_2=f$ and $A_f >> A_s$ and A_o, then $J_s(\vec{\alpha}_1;\vec{\alpha}_2)=I_s(\vec{\alpha}_1)\delta(\vec{\alpha}_1-\vec{\alpha}_2)$ and

$$J_o(\Delta\vec{\xi})=\frac{1}{(\bar{\lambda} f)^2}\int_{\Sigma_1} I_s(\vec{\alpha}) e^{j\frac{2\pi}{\bar{\lambda} f}\vec{\alpha}\cdot\Delta\vec{\xi}}\, dA_{\vec{\alpha}} \quad where\ \Delta\vec{\xi}=\vec{\xi}_2-\vec{\xi}_1.$$

(i) In the incoherent limit

For total incoherence of the object illumination $J_o(\Delta\vec{\xi})=I_o\cdot\delta(\Delta\vec{\xi})$,

$I_i(\vec{u})=I_o\int_{\Sigma_1}|K(\vec{u}-\vec{\xi})|^2\cdot|t_o(\vec{\xi})|^2\, dA_{\vec{\xi}}$, which is a convolution of the object intensity transmittance $|t_o(\vec{\xi})|^2$ with an intensity point spread function $|K(\vec{u}-\vec{\xi})|^2$. Note that $J_o(\Delta\vec{\xi})=I_o\cdot\delta(\Delta\vec{\xi}) \overset{FT}{\Rightarrow} \tilde{J}_o(\vec{P})=FT[J_o]=I_o$, therefore the spatial spectrum of the image $\tilde{I}_i(\vec{v})=FT[J_i(\vec{u}_1=\vec{u}_2=\vec{u})]$ becomes

$$\tilde{I}_i(\vec{v})=I_o\cdot FT[|t_o(\vec{\xi})|^2]FT[|K(\vec{u})|^2]=I_o\cdot\int_{\Sigma_o}\tilde{t}_o(\vec{z}_1)\tilde{t}_o^*(\vec{z}_1-\vec{v})\, dA_{\vec{z}_1}\cdot\int_{\Sigma_p}\tilde{K}(\vec{P}')\tilde{K}^*(\vec{P}'-\vec{v})\, dA_{\vec{P}'}.$$

A normalized form of $\int_{\Sigma_p} \tilde{K}(\vec{P})\tilde{K}^*(\vec{P}-\vec{v})\, dA_{\vec{P}}$ is identified to be the optical transfer function (OTF) of the imaging system

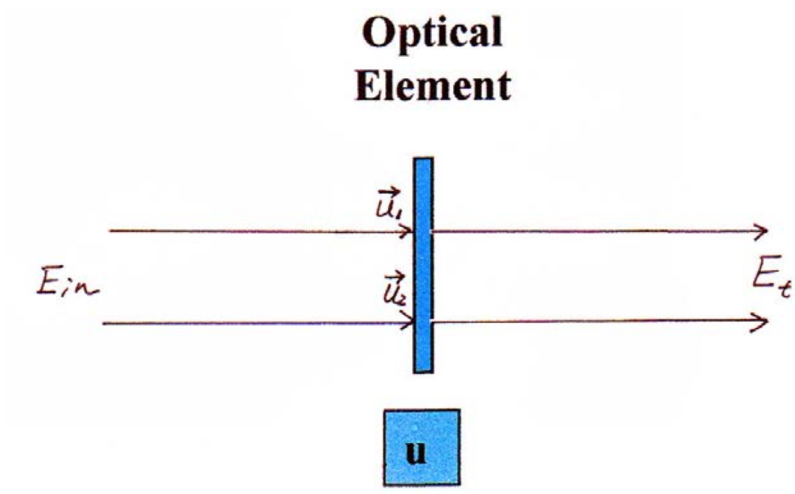
$$H(\vec{v}) = OTF = \frac{\int_{\Sigma_p} \tilde{K}(\vec{P})\tilde{K}^*(\vec{P}-\vec{v})dA_{\vec{P}}}{\int_{\Sigma_p} \left|\tilde{K}(\vec{P})\right|^2 dA_{\vec{P}}} = \frac{\int_{\Sigma_p} P_c(\vec{P})P_c^*(\vec{P}-\bar{\lambda} z_i \vec{v})dA_{\vec{P}}}{\int_{\Sigma_p} \left|P_c(\vec{P})\right|^2 dA_{\vec{P}}}.$$

(ii) In the coherent limit

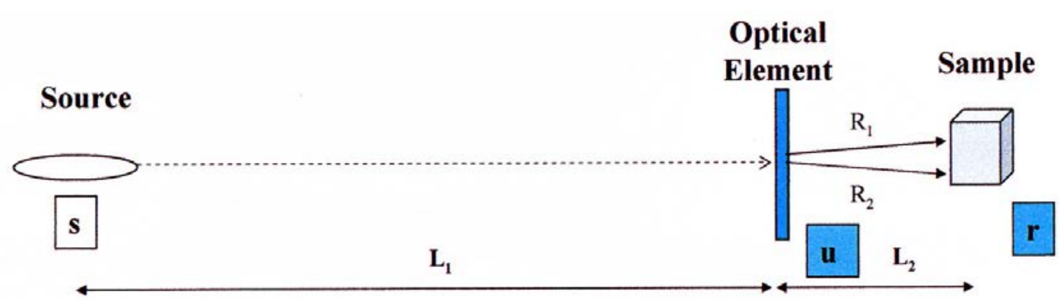
From $J_o(\Delta\vec{\xi}) = I_o$, $I_i(\vec{u}) = I_o \left|\int_{\Sigma} K(\vec{u}-\vec{\xi}) \cdot t_o(\vec{\xi})dA_{\vec{\xi}}\right|^2 = A_i(\vec{u})A_i^*(\vec{u})$, where

$A_i(\vec{u}) = \sqrt{I_o} \cdot \int_{\Sigma} K(\vec{u}-\vec{\xi}) \cdot t_o(\vec{\xi})dA_{\vec{\xi}}$ is a convolution of the amplitude spread function $K(\vec{u}-\vec{\xi})$ with the amplitude transmittance $t_o(\vec{\xi})$.

To summarize the results, refer to a schematic diagram shown below:



For an optical element with an amplitude transmittance $t_o(\vec{u})$, the transmitted amplitude can be expressed as: $E_t(\vec{u}, t) = t_o(\vec{u}) E_{in}(\vec{u}, t-\tau)$. The corresponding transmitted MIF becomes $J_t(\vec{u}_1, \vec{u}_2) = t_o(\vec{u}_1) t_o^*(\vec{u}_2) J_{in}(\vec{u}_1, \vec{u}_2)$.



For a sample illuminated by a source through an optical element, the MIF at the sample position can be derived to be

$$\boxed{J_{\mathbf{o}}(\vec{r}_1, \vec{r}_2) = \int_{\Sigma} d\vec{u}_1 \int_{\Sigma} d\vec{u}_2\, t_o(\vec{u}_1) t_o^*(\vec{u}_2) J_{in}(\vec{u}_1, \vec{u}_2) G_{L_2}(\vec{r}_1 - \vec{u}_1) G_{L_2}^*(\vec{r}_2 - \vec{u}_2)},$$

where

$G_{L_2}(\vec{r} - \vec{u}) = \dfrac{1}{j\lambda L_2} e^{jk(\vec{r}-\vec{u})^2/L_2}$ is the Green's function (i.e., the free-space propagator).

In general, coherence properties of the beam can be modified when pass through an optical element.

For a coherent illumination: $|\mu_{in}(\vec{u}_1, \vec{u}_2)| = 1$, the MIF can be factorized into $J_{in}(\vec{u}_1, \vec{u}_2) = \sqrt{I_{in}(\vec{u}_1)}\sqrt{I_{in}(\vec{u}_2)}\, e^{j(\alpha_1 - \alpha_2)}$. The MIF at the sample position can then be derived as

$$J_S^{coh}(\vec{r}_1, \vec{r}_2) = E_S(\vec{r}_1) \cdot E_S^{\ *}(\vec{r}_2) \text{ with } E_S(\vec{r}) = \int_\Sigma d\vec{u}\, t_o(\vec{u}) \sqrt{I_{in}(\vec{u})}\, G_{L_2}(\vec{r} - \vec{u}).$$

The coherence is found to be preserved since $|\mu_{in}(\vec{r}_1, \vec{r}_2)| = 1$.

For an incoherent illumination $J_{in}(\vec{u}_1, \vec{u}_2) = \kappa I_{in}(\vec{u}_1)\, \delta(\vec{u}_2 - \vec{u}_1)$. The MIF at the sample position can then read

$$J_S^{incoh}(\vec{r}_1, \vec{r}_2) = \frac{\kappa e^{-j\Psi}}{(\bar{\lambda} L_2)^2} \int_\Sigma d\vec{u} \left| t_o(\vec{u}) \right|^2 I_{in}(\vec{u}) e^{j2\pi(\vec{r}_2 - \vec{r}_1)\cdot\vec{u}/(\bar{\lambda} L_2)} \quad ,$$

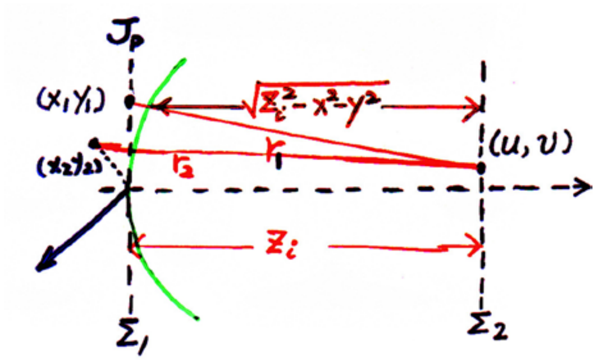
implying a phase object located just behind an incoherent light source will not change the coherence of the optical beam.

8.3 Image Formation is an Interferometric Process

In this section, we will discuss an interesting concept that views image formation as an interferometric process. By using this concept, novel means for gathering image data can be developed.

8.3.1 Why imaging can be viewed as an interfering process?

Consider the mutual intensity function $J_p(\vec{x}_1, \vec{x}_2)$ developed on the exit pupil Σ_1 -plane of an optical imaging system. We can simulate the imaging function by placing an effective lens at the principal plane as illustrated in the following diagram:

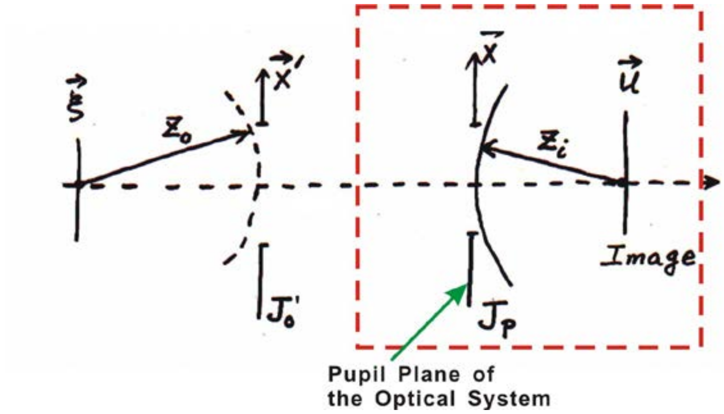


By using the technique developed in the previous section, the mutual intensity function $J_i(\vec{u}_1, \vec{u}_2)$ on the Σ_2 -plane can be related to $J_p(\vec{x}_1, \vec{x}_2)$ on the Σ_1 -plane. The image distribution can then be expressed as

$$I_i(\vec{u}) = J_i(\vec{u}_1 = \vec{u}_2 = \vec{u}) = \frac{1}{(\bar{\lambda} z_i)^2} \int_{\Sigma_1} dA_{\vec{x}_1} \int_{\Sigma_1} dA_{\vec{x}_2} J_p{}'(\vec{x}_1, \vec{x}_2)\, e^{j\frac{2\pi}{\bar{\lambda} z_i}(\vec{x}_2 - \vec{x}_1)\cdot\vec{u}} .$$

The equation shown above indicates that the intensity on the image plane is simply the Fourier transform of the mutual intensity on the exit pupil. The mutual intensity on the exit pupil can be regarded as consisting of a multitude pairs of pinholes with correlation specified by $J_p\,'(\vec{x}_1, \vec{x}_2)$.

Note also that a light source used to illuminate the object can produce a mutual intensity function $J_o^{'}$ directly behind the object plane



.

Assuming the object is incoherently illuminated with $\tilde{J}_o = FT[J_o(\Delta\vec{\xi})\delta(\Delta\vec{\xi})] = I_o$, the image spectrum can then be written as

$$\begin{aligned}\tilde{I}_i(\vec{\nu}) &= I_o \int \tilde{t}_o(\vec{z})\tilde{t}_o^{\,*}(\vec{z}-\vec{\nu})\tilde{K}(\vec{z})\tilde{K}^*(\vec{z}-\vec{\nu})\, dA_{\vec{z}} \\ &= \int \tilde{A}_i(\vec{z})\tilde{A}_i^{\,*}(\vec{z}-\vec{\nu})\, dA_{\vec{z}}\end{aligned} ,$$

where $\tilde{A}_i(\vec{z}) = FT[A_i(\vec{u})]$ is an image amplitude spectrum given by an Amplitude Transfer Function (ATF) $\tilde{K}(\vec{\nu}) = P(\lambda z_i \vec{\nu})$ of the optical system.

Let us consider an object of two closely spacing points along the ξ-axis

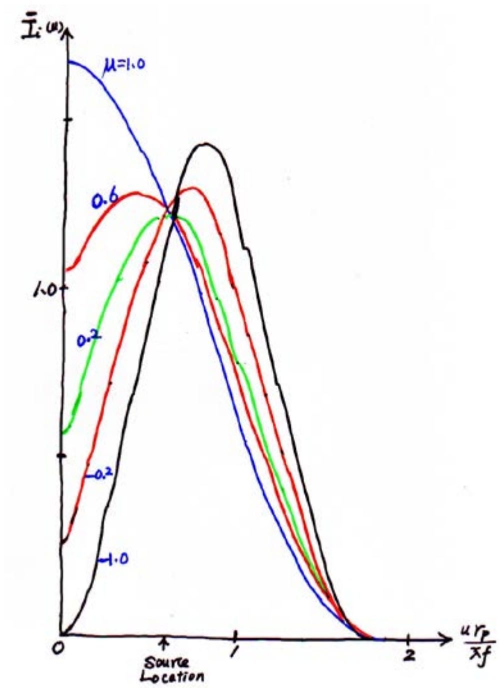
$$t_o(\vec{\xi}) = a\,\delta(\xi - \frac{S}{2}, \eta) + a\,\delta(\xi + \frac{S}{2}, \eta) .$$

The image intensity formed by the optical system with an amplitude point spread function $K(\vec{u}) = \dfrac{1}{\bar{\lambda} f} \int_{\Sigma} dA_{\vec{x}} P(\vec{x})\, e^{-j\frac{2\pi}{\bar{\lambda} f}(\vec{u}\cdot\vec{x})}$ can be expressed as

$$
\begin{aligned}
I_i(\vec{u}) &= \int_{\Sigma} dA_{\vec{\xi}} \int_{\Sigma} dA_{\vec{\xi}+\Delta\vec{\xi}} K(\vec{u}-\vec{\xi}) K^*(\vec{u}-\vec{\xi}-\Delta\vec{\xi}) t_o(\vec{\xi}) t_o^*(\vec{\xi}+\Delta\vec{\xi}) J_o(\Delta\vec{\xi}) \\
&= I_1 \left\{ \left| K(u-\frac{S}{2}, v) \right|^2 + \left| K(u+\frac{S}{2}, v) \right|^2 + 2\,\mathrm{Re}\left[\mu\, K(u-\frac{S}{2}, v) K^*(u+\frac{S}{2}, v) \right] \right\},
\end{aligned}
$$

where $I_1 = a^2 J_o(0,0), \quad \mu = \dfrac{a^2}{I_1} J_o(S,0)$.

For a pair of circular apertures with radius r_p,



$$
I_i(u) = I_1 \left\{ K^2(u-\frac{S}{2}) + K^2(u+\frac{S}{2}) + 2\,\mu K(u-\frac{S}{2}) K(u+\frac{S}{2}) \cos\phi \right\}
$$

and

$$
K(u) = 2\frac{\pi r_p^{\,2}}{\bar{\lambda} f} \left\{ \frac{J_1[2\pi r_p \rho / (\bar{\lambda} f)]}{[2\pi r_p \rho / (\bar{\lambda} f)]} \right\}.
$$

This results in an OTF of

$$
H(\vec{\nu}) = \int_{\Sigma_p} P(\vec{x}) P^*(\vec{x} - \bar{\lambda} z_i \vec{\nu})\, dA_{\vec{x}} \Big/ \int_{\Sigma_p} \left| P(\vec{x}) \right|^2 dA_{\vec{x}} .
$$

Optical aberration will introduce a spatial phase $\cos\phi$ for a pinhole pair with an identical spacing vector but at different positions. Thus, let us use a pair of pinholes separated by S to gather the image information with an interferometric process from the two pinholes. By varying the *spacing* and *orientations* of pinhole pairs, the entire image can be constructed from the complete set of interferograms.

8.3.2 Gathering Image Information with Interferometers

From the above discussion, we can form an image fringe from a single pair of pinholes on **the exit pupil plane** with a spacing vector of $\bar{\lambda} z_i \vec{\nu}$. For an illuminated object

$$J_p(\vec{x}_1, \vec{x}_2 = \vec{x}_1 - \bar{\lambda} z_i \vec{\nu}) \quad \Rightarrow \quad J_o[M\,\vec{x}_1, M\,(\vec{x}_1 - \bar{\lambda} z_i \vec{\nu})] \quad \Rightarrow \quad J_o \cdot FT[I_o].$$

From van Citter-Zernike Theorem, **different pinhole pairs with the same spacing vector yield an identical pattern of fringes**. Based on the concept of the Young's interference experiment, the spatial frequency component $\vec{\nu} = (\nu_U, \nu_V)$ of an image can arise from a pair of pinholes with a separation of $\Delta\vec{x} = (\vec{x}_2 - \vec{x}_1) = \bar{\lambda} z_i \vec{\nu}$. Thus, the observed image intensity distribution $I_i(\vec{u})$ can be viewed as being built up of a multitude of fringes generated by all possible pairs of pinholes with the fringes' amplitude and phase determined by the mutual intensity $J_p\,'(\vec{x}_1, \vec{x}_2)$. The Fourier spectrum of the image $I_i(\vec{u})$ becomes

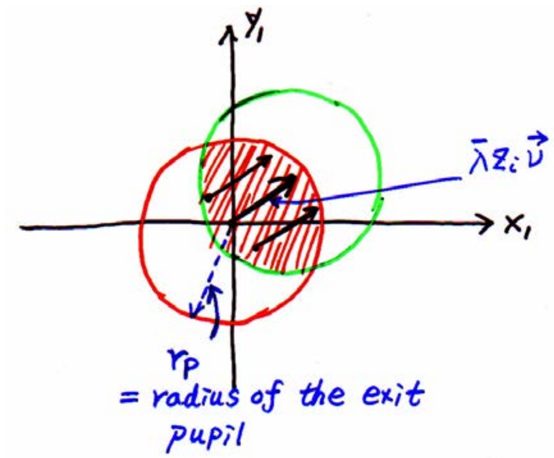
$$\boxed{\begin{aligned}\tilde{I}_i(\vec{\nu}) &= \int dA_{\vec{u}}\; I_i(\vec{u}) e^{j2\pi\vec{u}\cdot\vec{\nu}} \\ &= \frac{1}{(\bar{\lambda} z_i)^2} \int \underline{dA_{\vec{u}}\; e^{j2\pi\vec{u}\cdot\vec{\nu}}} \cdot \int_{\Sigma_p} dA_{\vec{x}_2} \int_{\Sigma_p} dA_{\vec{x}_1}\; J_p\,'(\vec{x}_1, \vec{x}_2)\, \underline{e^{j\frac{2\pi}{\bar{\lambda} z_i}\vec{u}\cdot(\vec{x}_1 - \vec{x}_2)}} \\ &= \int_{\Sigma_p} dA_{\vec{x}_1} \int_{\Sigma_p} dA_{\vec{x}_2}\; J_p\,'(\vec{x}_1, \vec{x}_2)\, \underline{\delta(\vec{\nu} + \frac{\vec{x}_2 - \vec{x}_1}{\bar{\lambda} z_i})} \\ &= \int_{\Sigma_p} dA_{\vec{x}_1}\; J_p\,'(\vec{x}_1, \vec{x}_1 - \bar{\lambda} z_i \vec{\nu})\end{aligned}}\;.$$

That is: the image spectrum at $\vec{\nu} = (\nu_U, \nu_V)$ originates from an integration of $J_p(\vec{x}_1, \vec{x}_1 - \bar{\lambda} z_i \vec{\nu})$ with respect to $\vec{x}_1$ over the pupil plane for a fixed separation $\bar{\lambda} z_i \vec{\nu}$.

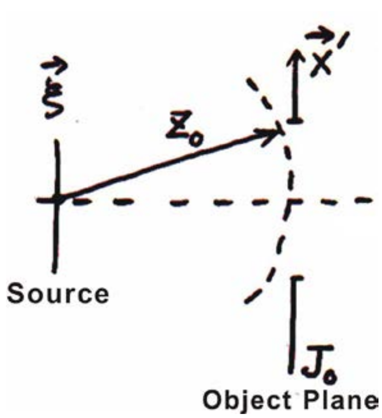
Therefore, adding all Young's fringe patterns generated by all pinhole pairs of spacing $\bar{\lambda} z_i \vec{\nu}$ can form the image.

Note $J_p{}'(\vec{x}_1, \vec{x}_2) = P(\vec{x}_1) P^*(\vec{x}_2) J_p(\vec{x}_1, \vec{x}_2)$ with $P(\vec{x}_1)$ determined by the bounds of the exit pupil, apodization, and aberrations, results in an image spectrum of

$$\tilde{I}_i(\vec{\nu}) = FT[I_i(\vec{u})] = \int_{\Sigma_1} dA_{\vec{x}_1}\ P(\vec{x}_1) P^*(\vec{x}_1 - \bar{\lambda} z_i \vec{\nu}) J_p(\vec{x}_1, \vec{x}_1 - \bar{\lambda} z_i \vec{\nu}) .$$



Consider the case of an incoherently illuminated object



From van Citter-Zernike theorem,

$$J_o(\vec{x}_1, \vec{x}_2) = \frac{1}{(\bar{\lambda} z_o)^2} \int dA_{\vec{\xi}}\ I_s(\vec{\xi})\, e^{j\frac{2\pi}{\bar{\lambda} z_o} \vec{\xi} \cdot (\vec{x}_2 - \vec{x}_1)} = J_o(\Delta \vec{x} = \vec{x}_2 - \vec{x}_1) .$$

By using $J_p(\vec{x}_1, \vec{x}_2) = J_o(M\vec{x}_1, M\vec{x}_2) = J_o(M\Delta\vec{x})$, we can obtain

$$\tilde{I}_i(\vec{\nu}) = \int_{\Sigma_1} dA_{\vec{x}_1}\ P(\vec{x}_1)P^*(\vec{x}_1 - \bar{\lambda} z_i \vec{\nu}) J_p(\bar{\lambda} z_i \vec{\nu}) = J_p(\bar{\lambda} z_i \vec{\nu}) \cdot \int_{\Sigma_1} dA_{\vec{x}_1}\ P(\vec{x}_1)P^*(\vec{x}_1 - \bar{\lambda} z_i \vec{\nu}),$$

which implies that for an incoherently illuminated object and an aberration-free optical system, the influence of the optical system on the image spectrum at $\vec{\nu}$ is simply the autocorrelation integral of two exit pupils displaced by $\bar{\lambda} z_i \vec{\nu}$.

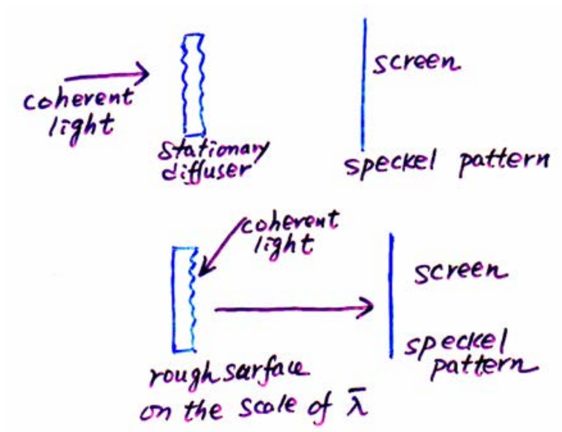
The redundancy of the optical system (*i.e.*, the multitude of ways a single spacing vector is embraced by the pupil) serves to increase *S*/*N* of the measurement but does not contribute new information. However, if the optical system is affected by aberrations, the redundancy can degrade the image quality by introducing different spatial phases, which will reduce the contrast of the resultant fringe.

The concept of interferometric image formation had already been employed to build an optical system with an effective aperture larger than the real imaging lens (or mirror). One of the examples is the technique of synthetic aperture imaging. See for example: **Sensors** 2008, 8, 3903-3931: Brynmor J. Davis, Daniel L. Marks, Tyler S. Ralston, P. Scott Carney and Stephen A. Boppart, *Interferometric Synthetic Aperture Microscopy: Computed Imaging for Scanned Coherent Microscopy*.

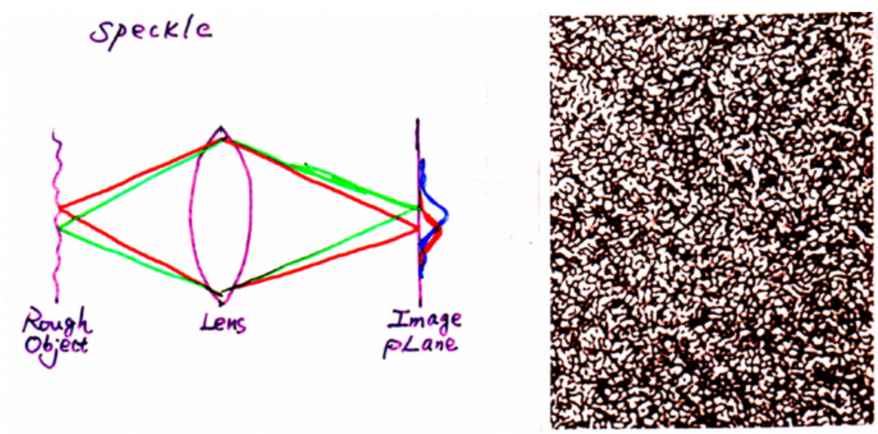
An interesting imaging modality with structured light illumination, which can conquer the resolution limit imposed by Abbe-Rayleigh criterion, can be found in **Biophysical Journal** 2008, 94, 4957–4970: Mats G. L. Gustafsson,Lin Shao,y Peter M. Carlton,y C. J. Rachel Wang,Inna N. Golubovskaya, W. Zacheus Cande, David A. Agard, and John W. Sedat, *Three-Dimensional Resolution Doubling in Wide-Field Fluorescence Microscopy by Structured Illumination*.

8.4 The Speckle Effect in Coherent Imaging

In the experimental situation of a coherent light wave passing through a diffuser or reflecting from a rough object (see the diagram shown in the following),



the lack of knowledge of the detailed microscopic structure of the complex object wave raises a need to discuss the properties of speckle in statistical terms.



The spatial distribution in a speckle pattern is sufficiently complicated to be described by a statistically stationary and ergodic random process. Stationarity requires the statistical properties of an ensemble of speckle patterns to be the same as those of an individual speckle pattern within the ensemble. Ergodicity requires the statistical properties of two spatial positions to be independent and identical to those of the ensemble.

8.4.1 The origin and 1st-order statistics of speckle

The immediate issue we met is how to depict the complex light field (such as speckle). We can describe the light field as

■ *an ensemble of field distributions* with the same macroscopic properties but differing in microscopic detail.

This leads to a wavefront with *phases* reflecting individual contributions from a rough surface on the scale of $\overline{\lambda}$.

Thus, it is appropriate to assume the wavefront to be

■ *linearly polarized thermal light* with an intensity probability density distribution (i.e., Rayleigh PDF)

$$P_I(I) = \begin{cases} \frac{1}{\overline{I}} e^{-I/\overline{I}}, & I \geq 0 \\ 0\,, & otherwise \end{cases}.$$

The contrast of a speckle pattern can be defined as

$$C = \frac{\sqrt{\int_0^\infty (I - \overline{I})^2 P_I(I)\, dI}}{\overline{I}}.$$

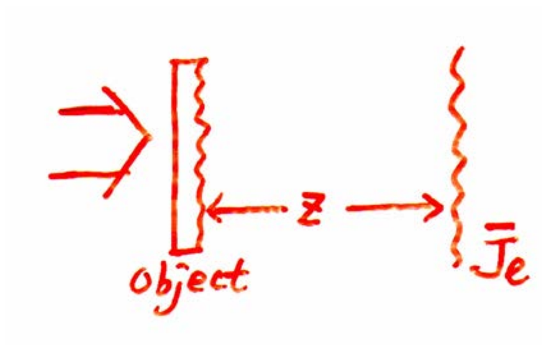
In this speckle pattern, $C = 1$ implies the intensity fluctuates rather pronounced.

8.4.2 Ensemble Average Coherence

Resulting from the randomness of complex object light, we shall calculate an ensemble average of the mutual intensity function.

Consider an ensemble of ideally rough surface profiles, the mutual intensity function of an optical field from the surface becomes

$J(\vec{\xi}_1, \vec{\xi}_2) = K\ \overline{I}(\vec{\xi}_1)\delta(\vec{\xi}_1 - \vec{\xi}_2)$, where $\overline{I}(\vec{\xi}_1)$ is the ensemble-averaged intensity distribution across the rough object.



After propagating a distance of *z*=*l*, the resulting MIF (i.e., the spatial intensity correlation function $C_I(\Delta r)$) can be expressed as

$$\overline{J}_l(\vec{x}_1, \vec{x}_2) = \frac{Ke^{-j\psi}}{(\overline{\lambda}l)^2} \int_{Object\ Plane} \overline{I}(\vec{\xi})\ e^{j\frac{2\pi}{\overline{\lambda}l}\vec{\xi}\cdot(\vec{x}_2 - \vec{x}_1)} dA_{\vec{\xi}}\ .$$

8.5 Controlled Generation of Complex Light (ref: Nicholas Bender, Hasan Yılmaz, Yaron Bromberg, and Hui Cao, "Creating and Controlling Complex Light", APL Photonics 4, 110806 (2019))

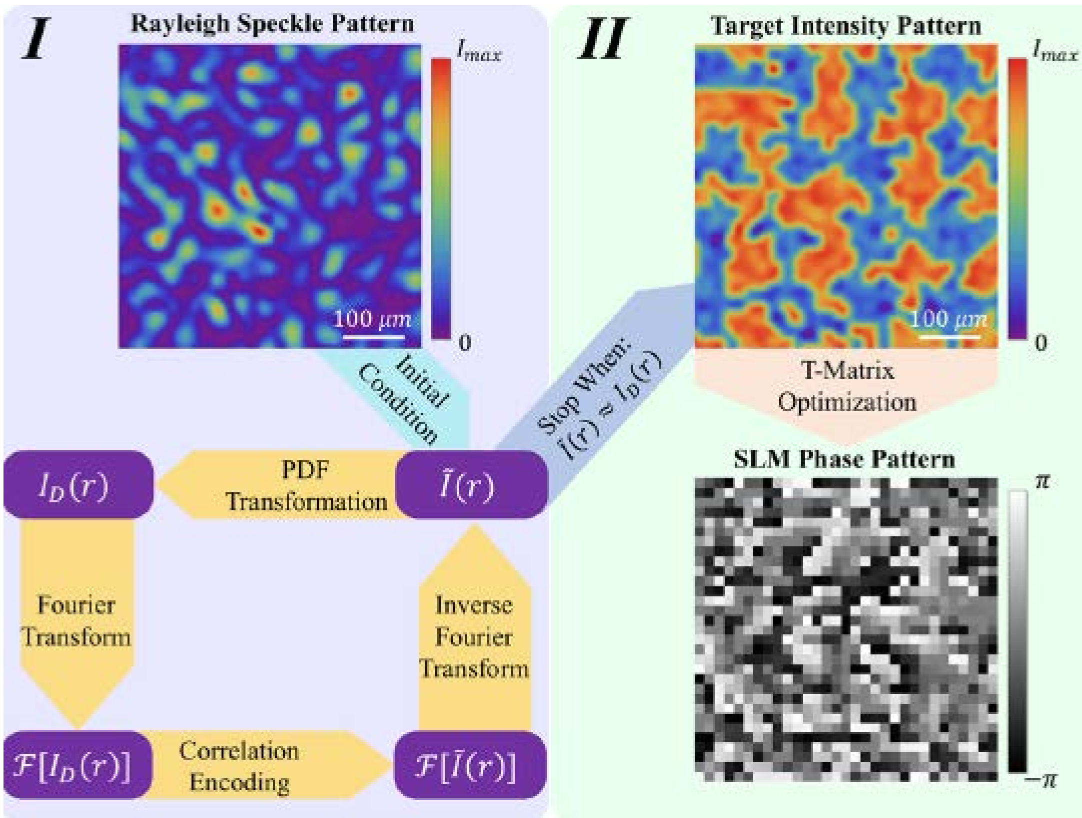
The ability to independently control the intensity PDF and correlations of speckles has many potential applications. For example, they can be used as a form of 'smart' illumination in high-order ghost imaging, dynamic speckle illumination microscopy, super-resolution imaging, compressive sensing, and optical sectioning microscopy.

In a speckle pattern, the spatial field correlation function is defined as $C_E(\Delta r) = \left\langle E(r)E^*(r+\Delta r)\right\rangle \big/ \left\langle |E(r)|^2 \right\rangle$. The spatial intensity correlation function is given by:

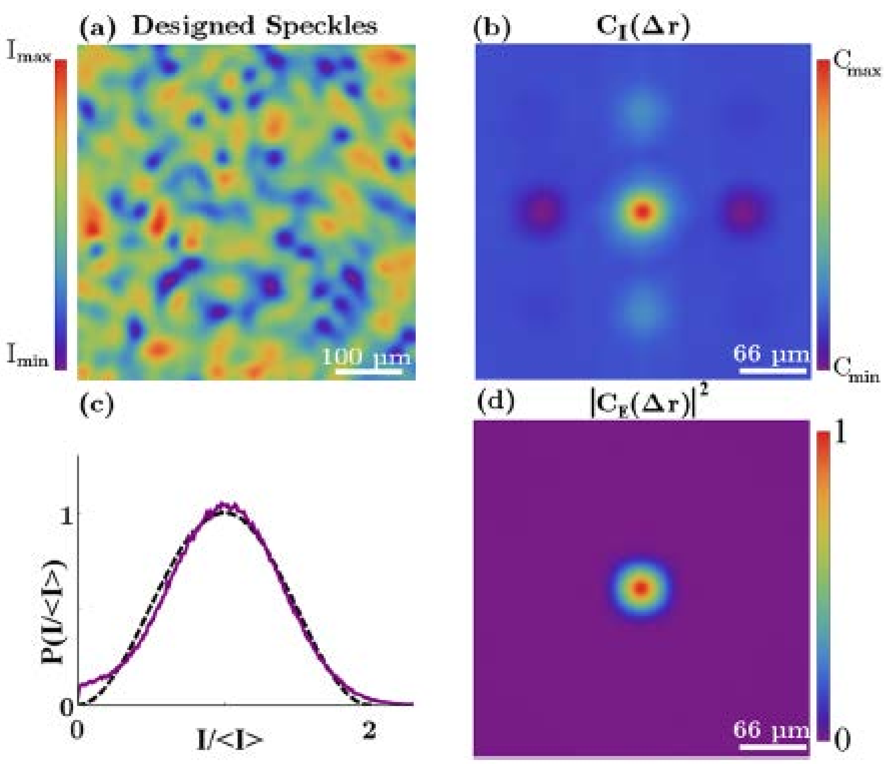
$$\begin{aligned} C_I(\Delta r) &= \left\langle I(r)I^*(r+\Delta r)\right\rangle \big/ (\left\langle I(r)\right\rangle \left\langle I(r+\Delta r)\right\rangle) - 1 \\ &= C_L(\Delta r) + C_{NL}(\Delta r) \end{aligned}.$$

Here $C_L(\Delta r)$ is known as the local correlation function, and it is related to the field correlation function by $C_L(\Delta r) = C_0 |C_E(\Delta r)|^2$, where $C_0 = \left\langle I^2 \right\rangle \big/ \left\langle I \right\rangle^2 - 1$ is the speckle contrast. $C_{NL}(\Delta r)$ represents the non-local correlation function.

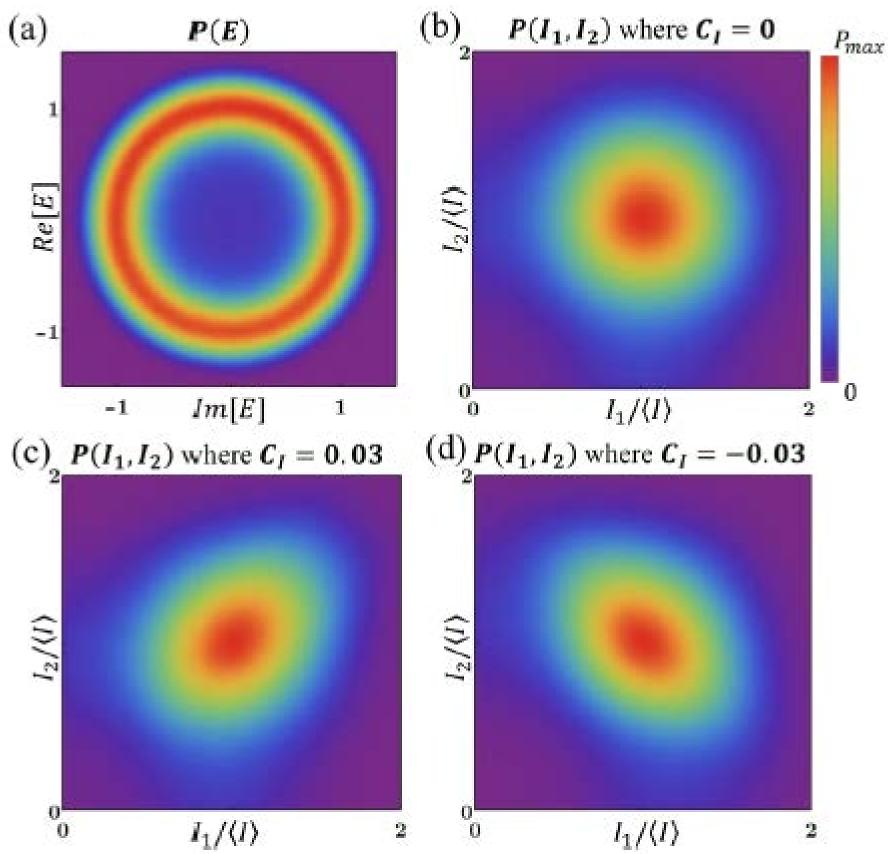
This paper experimentally demonstrates a method of simultaneously customizing the intensity PDF of speckle patterns and spatial correlations among the speckle grains. Various families of speckles are created by encoding high-order correlations into the phase front of a monochromatic laser beam with a spatial light modulator (SLM).



I
Rayleigh Speckle Pattern
I_{max}
0
100 μm
II
Target Intensity Pattern
I_{max}
0
100 μm
Initial Condition
Stop When: $\tilde{I}(r) \approx I_D(r)$
T-Matrix Optimization
SLM Phase Pattern
π
$-\pi$
$I_D(r)$
PDF Transformation
$\tilde{I}(r)$
Fourier Transform
Inverse Fourier Transform
$\mathcal{F}[I_D(r)]$
Correlation Encoding
$\mathcal{F}[\tilde{I}(r)]$



(a) Designed Speckles
I_{max}
I_{min}
100 μm
(b) $C_I(\Delta r)$
C_{max}
C_{min}
66 μm
(c)
P(I/<I>)
1
0
0
2
I/<I>
(d) $|C_E(\Delta r)|^2$
1
0
66 μm



Complex light speckle technique had also been applied to image highly scattering biological tissues. Here speckles play a carrier of information about tissue microstructure. An interesting overview of speckle in optical coherence tomography can be found in http://biomedicaloptics.spiedigitallibrary.org/article.aspx?articleid=1101244.

This paper discusses the origin, statistical properties, and classification of speckle in OCT. The concepts of signal-carrying and signal-degrading speckle are defined in terms of the phase and amplitude disturbances of the sample beam. Four speckle-reduction methods—polarization diversity, spatial compounding, frequency compounding, and digital signal processing—were presented to reveal the potential effectiveness of each method with the aid of examples.