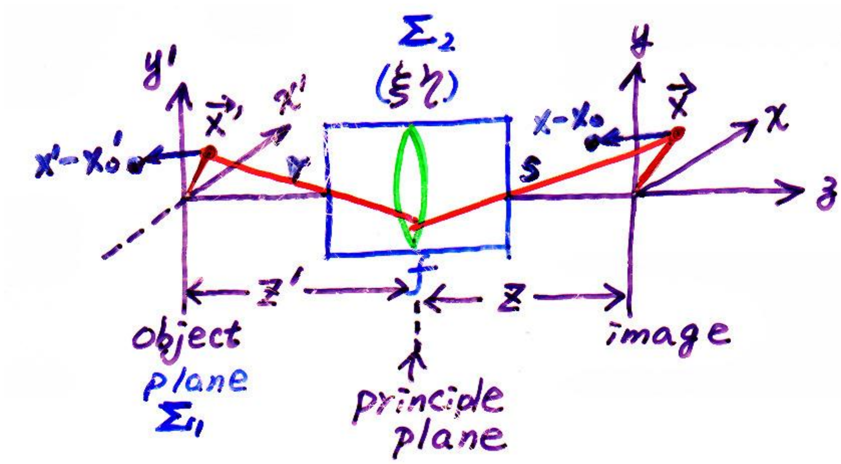
Chapter 6 Image Formation Theory

In this chapter, we will discuss the mathematical formalism of image formation process and relevant image resolution issue.

6.1 Intensity Impulse Response

First, let us consider an imaging system as depicted below. The object is illuminated by an incoherent light source.



Since Maxwell's equations, which govern the propagation of light in free space, are linear and the optical system is *stationary*, the following principles can be invoked to facilitate the description of image formation:

■ **Linearity:** Assuming $f_1(x')$ *and* $f_2(x')$ are two sources lying on the object plane and the optical system images $f_1(x')$ *into* $g_1(x)$, $f_2(x')$ *into* $g_2(x)$, respectively, then the system will produce $a\,g_1(x)+b\,g_2(x)$ for the linearly combined source $a\,f_1(x')+b\,f_2(x')$.

■ **Stationarity:** If the system maps $f_1(x') \rightarrow g_1(x)$, then it will also map $f_1(x'-x_0') \rightarrow g_1(x-x_0)$.

Based on the above properties, we can further draw the following conclusions:

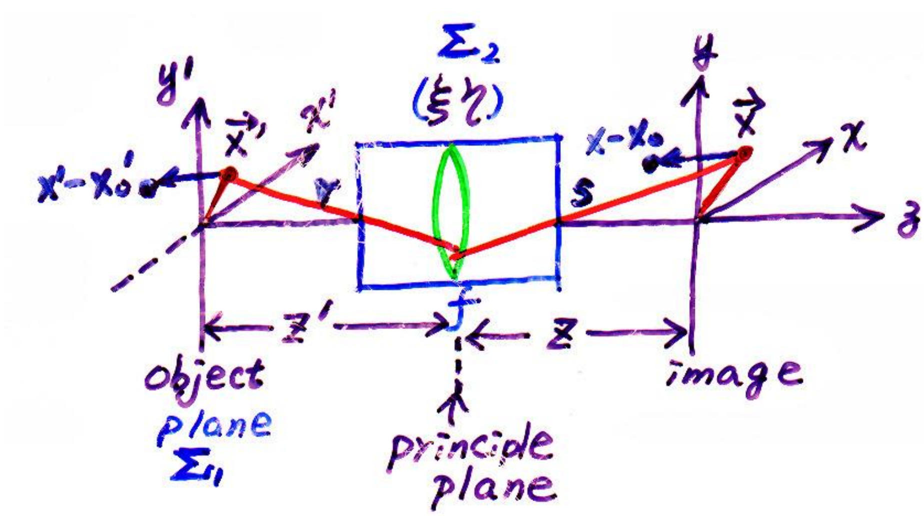
(i) Let $s(x)$ be the image for a point source on the object plane, i.e., $\delta(x') \rightarrow s(x)$, an arbitrarily distributed intensity distribution on the object plane can be expressed as $I_{ob}(x') = \int f(x'')\delta(x'-x'')\,dx''$.

(ii) From the linearity and stationarity of the system, the image distribution formed on the image plane will be $I_{im}(x) = \int f(x')s(x-x')\,dx'$.

Here $s(x)$ is called as the **point spread function** (*psf*), which is the intensity impulse response, or the point diffraction pattern of the optical system.

6.1.1 The Expression of Intensity Impulse Response

Refer to the figure



and recall that the effect of a thin lens is to impose a phase factor $\exp[-jk\xi^2/(2f)]$ on the incident wave with a transmission factor $A(\xi)$. Based on the Huygens-Fresnel principle, the optical field on the image plane $\vec{x}=(x,y)$ can be expressed as

$$U_{im}(x)=K_1\int_{\Sigma_1}\int_{\Sigma_2}U_{ob}(x')\frac{e^{jkr}}{r}\cdot A(\xi)e^{-\frac{jk\xi^2}{2f}}\cdot\frac{e^{jks}}{s}d\xi\,dx'.$$

By using the paraxial approximation at the far-field

$$\begin{aligned} r^2&=(\vec{x}'-\vec{\xi})\cdot(\vec{x}'-\vec{\xi})=R'^2-2\vec{x}'\cdot\vec{\xi}+|\vec{\xi}|^2\approx(R'-\frac{x'\xi}{R'}+\frac{\xi^2}{2R'})^2 \\ s^2&=(\vec{x}-\vec{\xi})\cdot(\vec{x}-\vec{\xi})=R^2-2\vec{x}\cdot\vec{\xi}+|\vec{\xi}|^2\approx(R-\frac{x\xi}{R}+\frac{\xi^2}{2R})^2 \end{aligned},$$

the optical field distribution at the image plane can be simplified to be

$$U_{im}(x)=K_1\frac{e^{jk(R+R')}}{RR'}\int_{\Sigma_1}\int_{\Sigma_2}U_{ob}(x')\cdot A(\xi)e^{-\frac{jk\xi^2}{2}(\frac{1}{R}+\frac{1}{R'}-\frac{1}{f})}\cdot e^{jk\xi(\frac{x'}{R'}+\frac{x}{R})}d\xi\,dx'.$$

With the paraxial approximation of $x'<<z'$ *and* $x<<z\approx f$, we achieve

$$U_{im}(x)=K\int_{\Sigma_1}\int_{\Sigma_2}U_{ob}(x')\cdot A(\xi)e^{-\frac{jk\xi^2}{2}(\frac{1}{z}+\frac{1}{z'}-\frac{1}{f})}\cdot e^{jk\xi(\frac{x'}{z'}+\frac{x}{z})}d\xi\,dx',$$

where $(\frac{1}{z}+\frac{1}{z'}-\frac{1}{f})=0$ on the image plane. And finally we have

$$U_{im}(x)=K\int_{\Sigma_2}\int_{\Sigma_1}U_{ob}(x')\cdot A(\xi)\cdot e^{jk\xi(\frac{x'}{z'}+\frac{x}{z})}dx'd\xi=K\int\tilde{U}_{ob}(\frac{\xi}{\lambda z'})A(\xi)\cdot e^{j2\pi(\frac{x}{\lambda z})\xi}d\xi,$$

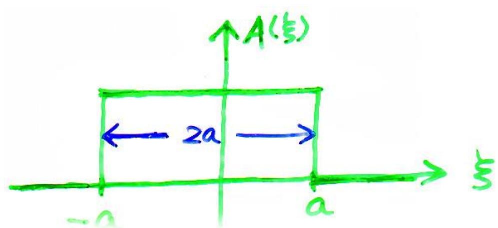
where $\tilde{U}_{ob}(\frac{\xi}{\lambda z'})=\int U_{ob}(x')e^{j2\pi(\frac{\xi}{\lambda z'})x'}dx'$.

For a point source of $u_{ob}(x') = \delta(x')$, $\tilde{U}_{ob}(\frac{\xi}{\lambda z'}) = \int \delta(x')e^{j2\pi(\frac{\xi}{\lambda z'})x'}dx' = 1$, we then obtain $U_{im}(x) = K\int_{\Sigma_2} A(\xi)\cdot e^{j2\pi(\frac{\xi}{\lambda z})x}d\xi$.

By defining $s(x) = \langle U_{im}(x)U_{im}^*(x)\rangle = \left|K\int_{\Sigma_2} A(\xi)\cdot e^{j2\pi(\frac{x}{\lambda z})\xi}d\xi\right|^2$, which indicates that the intensity impulse response $s(x)$ can be determined directly from the aperture transmission function $A(\xi)$, which is also called pupil function.

6.1.2 Image Formation in Terms of the Intensity Impulse Response

To facilitate further discussion, let us start with an 1-D case with a rectangular aperture transmission function $A(\xi)$



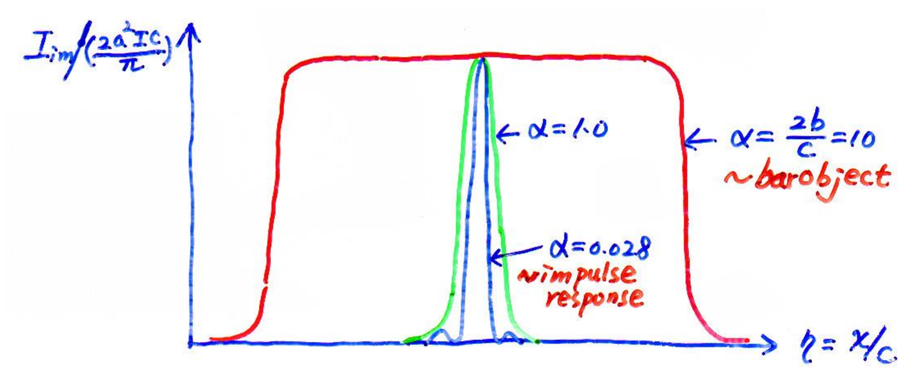
Based on the previous result, $s(x) = \left|\int_{-a}^{a} e^{j2\pi(\frac{\xi}{\lambda z})x}d\xi\right|^2 = 4a^2\cdot sinc^2(\frac{\pi a x}{\lambda f})$, indicating that when a bar object of $I_{ob}(x') = \begin{cases} I_o & |x'| \le b \\ 0 & |x'| > b \end{cases}$ is imaged, then its image will be

$$I_{im}(x) = \int s(x-x')I_{ob}(x')\,dx' = 4a^2 I_o\cdot\int_{x-b}^{x+b} sinc^2(\frac{\pi a x'}{\lambda f})\,dx'$$

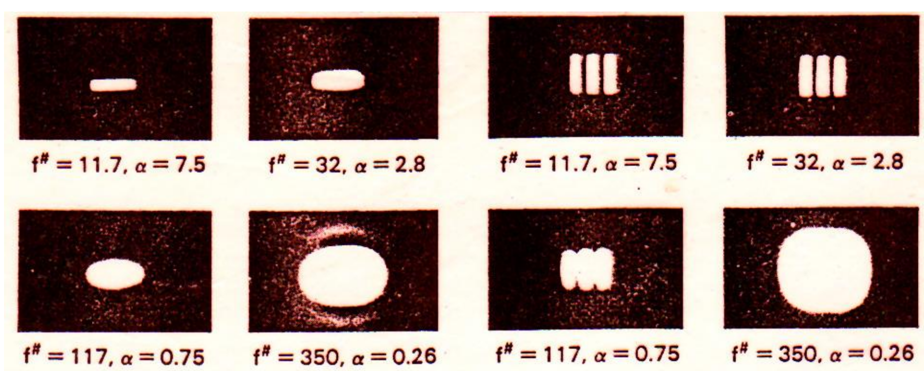
$$= 4a^2 I_o\cdot\frac{\lambda f}{2\pi a}\left\{\mathcal{S}(2\pi\frac{x+b}{C}) - \mathcal{S}(2\pi\frac{x-b}{C})\right\} = \frac{2a^2 C}{\pi}I_o\{\mathcal{S}_+ - \mathcal{S}_-\},$$

where C denotes the spot size of the intensity impulse response $C = 2\lambda \cdot \frac{f}{2a} = 2\lambda \cdot f^{\#}$, and $\mathscr{S}(x) = \int_0^x sinc^2(\frac{u}{2})\,du$.

Let us define $\alpha = 2b/C$ (the ratio of object dimension and image spot size)

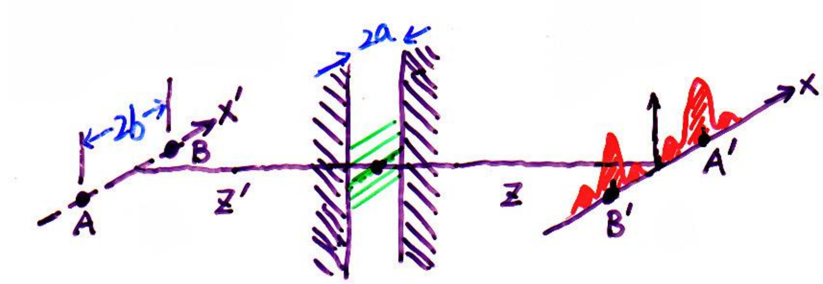


A series of images taking by an imaging apparatus with different $f^{\#}$ (i.e., different C) are shown below



6.1.3 Resolution in Terms of the Intensity Impulse Response

Now consider the image of two points $I_{ob}(x') = I_o \cdot [\delta(x'-b) + \delta(x'+b)]$ in 1-D case

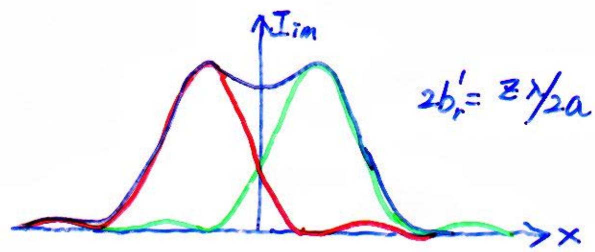


We calculate the image distribution

$$I_{im}(x) = 4a^2 I_o \cdot \int_{-\infty}^{+\infty} [\delta(x'-b) + \delta(x'+b)] sinc^2(\frac{ka(x-x')}{z})\, dx'$$
$$= 4a^2 I_o \cdot \left\{ sinc^2(\frac{ka(x-b)}{z}) + sinc^2(\frac{ka(x+b)}{z}) \right\},$$

where $s(x) = \left| \int_{-a}^{a} e^{j2\pi(\frac{x}{\lambda z})\xi} d\xi \right|^2 = 4a^2 \cdot sinc^2(\frac{kax}{z})$.

Rayleigh proposed a resolution criterion for an incoherent image by choosing a separation $2b_r'$ of the two *sinc*^{2} functions such that the central maximum of one coincides with the first minimum of the other. The resulting image intensity distribution has a small minimum at the center of the two-point images



,

with $2b_r' = \frac{z\lambda}{2a} = \lambda f^{\#} \quad if \;\; z = f$.

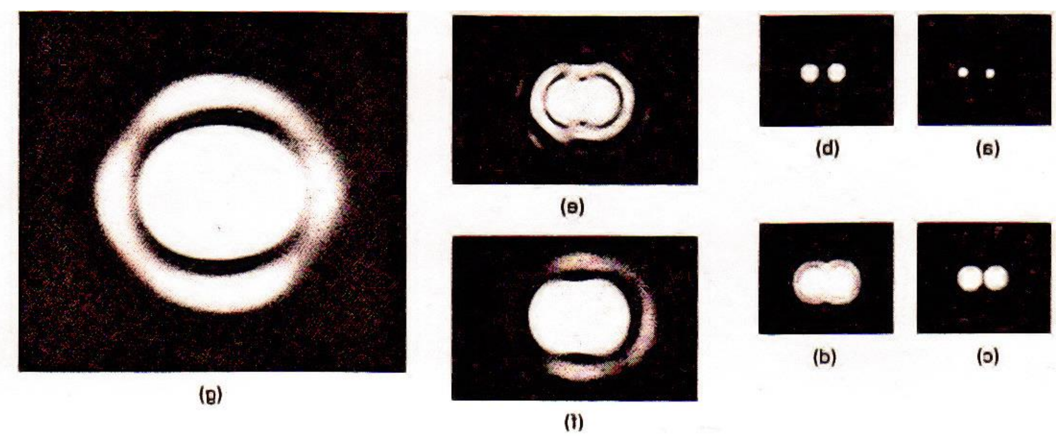
For partially coherent imaging, we can choose the separation of the two *sinc*^{2} functions such that $\frac{\partial^2}{\partial x^2} I_{im}(x)\big|_{x=0} = 0$. Note

$$\bar{I}_{im}(x) = \left\{ Sinc^2\left(\frac{ka(x-b)}{z}\right) + Sinc^2\left(\frac{ka(x+b)}{z}\right) \right\}$$ and $Sinc(x) = Sinc(-x)$. The resulting criterion (*i.e.*, the Sparrow criterion) then becomes

$$2b_s' = \frac{2.606}{\pi} \cdot \frac{z\lambda}{2a}.$$

For 2-D case, $s(x) = 4a^2[2J_1(\frac{kax}{z}) \Big/ (\frac{kax}{z})]^2$; where a is the aperture radius of the optical system. The corresponding resolution criteria become

$$2b_r' = 1.22 \cdot \frac{z\lambda}{2a}, \quad \textbf{or } 2b_s' = \frac{2.976}{\pi} \cdot \frac{z\lambda}{2a}.$$



6.2 Image Formation Described by the Intensity Transfer Function

Image formation described by the *Intensity Transfer Function* is usually more suitable for the imaging analysis of complicated objects. Consider an object with a cosinusoidal intensity distribution as $I_{ob}(x') = 1 + A\cos(2\pi\mu_o x')$, where A describes the modulation amplitude, and μ_o the spatial frequency of the object. In view that $s(x)$ is a real function, the image distribution formed on the image plane becomes

$$
\begin{aligned}
I_{im}(x) &= \int I_{ob}(x')s(x-x')dx' \\
&= \int [1+\frac{A}{2}(e^{2\pi j\mu_o x'}+e^{-2\pi j\mu_o x'})]\int \tilde{s}(\mu)e^{-j2\pi\mu(x-x')}d\mu\, dx' \\
&= \int [\delta(\mu)+\frac{A}{2}\delta(\mu-\mu_o)+\frac{A}{2}\delta(\mu+\mu_o)]\cdot \tilde{s}(\mu)e^{-j2\pi\mu x}d\mu \\
&= \tilde{s}(0)+\frac{A}{2}[\tilde{s}(\mu_o)e^{j2\pi\mu_o x}+\tilde{s}(-\mu_o)e^{-j2\pi\mu_o x}] \\
&= \tilde{s}(0)+A\cdot \mathrm{Re}[\tilde{s}(\mu_o)e^{j2\pi\mu_o x}] \\
&= \tilde{s}(0)\left[1+A\frac{|\tilde{s}(\mu_o)|}{\tilde{s}(0)}\cos(2\pi\mu_o x+\phi(\mu_o)\right]
\end{aligned}
$$

Visibility of the periodic image pattern can be displayed as

$$V = A\cdot\frac{|\tilde{s}(\mu_o)|}{\tilde{s}(0)} = A\cdot|H(\mu_o)|,$$

where $H(\mu_o)$ is the optical transfer function.

Consider a general object, which the object intensity distribution can be described by $I_{ob}(x)=\int \tilde{I}_{ob}(\mu)e^{-2\pi j\mu x}d\mu$, with $\tilde{I}_{ob}(\mu)$ being the object spectrum.

Then

$$
\begin{aligned}
I_{im}(x) &= \int \tilde{I}_{im}(\mu)e^{-j2\pi\mu x}d\mu \\
&= \int I_{ob}(x')s(x-x')dx' = \int [\int \tilde{I}_{ob}(\mu)e^{-j2\pi\mu x'}d\mu][\int \tilde{s}(\mu')e^{-j2\pi\mu'(x-x')}d\mu']dx' \\
&= \int \tilde{I}_{ob}(\mu)\cdot\tilde{s}(\mu)e^{-j2\pi\mu x}d\mu
\end{aligned}
$$

Thus, $\tilde{I}_{im}(\mu)=\tilde{I}_{ob}(\mu)\cdot\tilde{s}(\mu)$. Note $s(x)=\left|\int A(\xi)e^{-j2\pi\xi\frac{x}{\lambda z}}d\xi\right|^2 \quad \Rightarrow$

$$\begin{aligned}\tilde{s}(\mu) &= \int [\int A(\xi) e^{-j2\pi\xi\frac{x}{\lambda z}} d\xi][\int A^*(\xi') e^{j2\pi\xi'\frac{x}{\lambda z}} d\xi'] \cdot e^{j2\pi\mu x} dx \\ &= \int A^*(\xi') A(\xi' + \lambda z \mu)\, d\xi'\end{aligned},$$

implying that the un-normalized optical transfer function is given by the unnormalized autocorrelation of the aperture function with its complex conjugate.

6.2.2 Image Formation in Terms of the Optical Transfer Function

To facilitate further discussion on the optical transfer function, let us consider the following simple aperture function

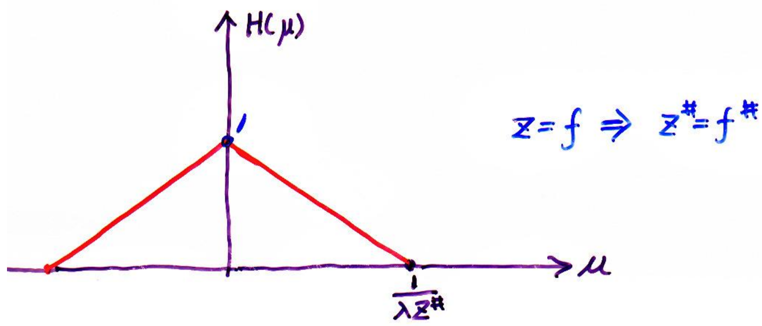
$$A(\xi) = \begin{cases} 1 & |\xi| \le a \\ \\ 0 & |\xi| > a \end{cases}.$$

$\tilde{s}(\mu) = \int A^*(\xi') A(\xi' + \lambda z \mu)\, d\xi'$ = the measure of the overlapping area as the aperture is slided across itself by $\lambda z \mu$: $\tilde{s}(\mu) = \int_{-a}^{a-\lambda z \mu} d\xi = 2a - \lambda z \mu = 2a[1 - \frac{\lambda z \mu}{2a}]$.

By normalizing $\tilde{s}(\mu)$, we deduce the optical transfer function to be

$$H(\mu) = \frac{|\tilde{s}(\mu)|}{\tilde{s}(0)} = (1 - \frac{\lambda z |\mu|}{2a}) \qquad as \ |\mu| \le \frac{2a}{\lambda f} = \frac{1}{\lambda f^{\#}}.$$

By defining $z^{\#} = \frac{z}{2a}$, $\Rightarrow \quad H(\mu) = (1 - \lambda z^{\#} |\mu|)$.



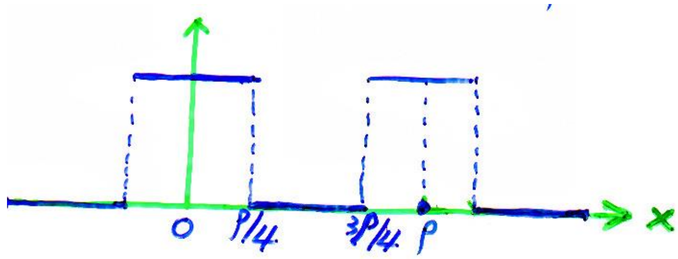
Some examples are given below for further illustration:

■ Image of a Ronchi Ruling

Consider an image of a Ronchi ruling with incoherent light, the ruling is described by

$$I_{ob}(x') = \begin{cases} 1 & x' < \rho/4 \\ 0 & \rho/4 < x' < 3\rho/4 \\ 1 & 3\rho/4 < x' < \rho \end{cases}$$

and $I_{ob}(x' \pm \rho) = I_{ob}(x')$.

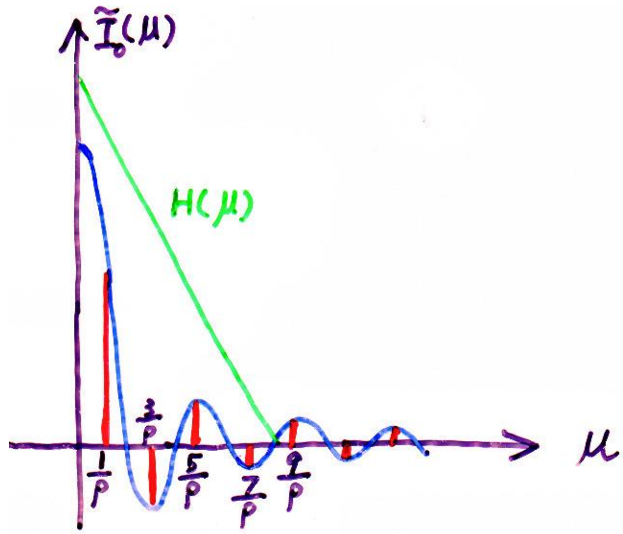


Then, $I_{im}(x) = \int \tilde{I}_{im}(\mu) e^{-j2\pi\mu x} d\mu = \int I_{ob}(x') s(x-x') dx'$, where $\tilde{I}_{im}(\mu) = \tilde{I}_{ob}(\mu) \cdot H(\mu)$.

For the object with Ronchi ruling

$$\tilde{I}_{ob}(\mu) = \int_{-\infty}^{\infty} I_{ob}(x') e^{2\pi j\mu x'} dx' = \frac{\rho}{4} sinc[\frac{2\pi\rho\mu}{4}] \cdot \sum_{n=-\infty}^{\infty} e^{2\pi j\mu\rho \cdot n} = \frac{\rho}{4} sinc[\frac{2\pi\rho\mu}{4}] \cdot \sum_{n=-\infty}^{\infty} \delta(\mu - \frac{n}{\rho})$$

and $H(\mu) = (1 - \lambda z^{\#} |\mu|)$ for $|\mu| \le 1/(\lambda z^{\#})$.

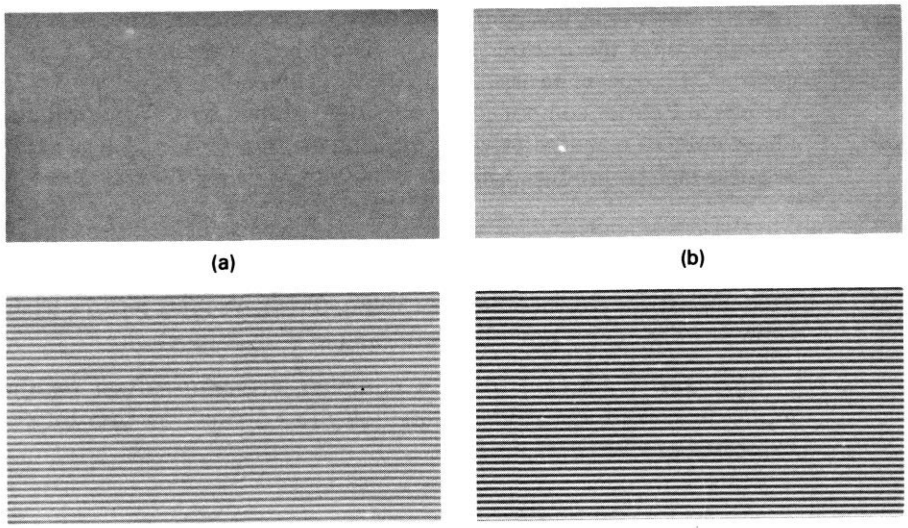


A. If $\dfrac{1}{\rho} > \dfrac{1}{\lambda z^{\#}}$, only dc terms of the ruling passes through the optical system described by $H(\mu)$, resulting in no intensity variation.

B. If $\dfrac{1}{\rho} < \dfrac{1}{\lambda z^{\#}} < \dfrac{3}{\rho}$, the resultant image consists of the central order of the *sinc* function.

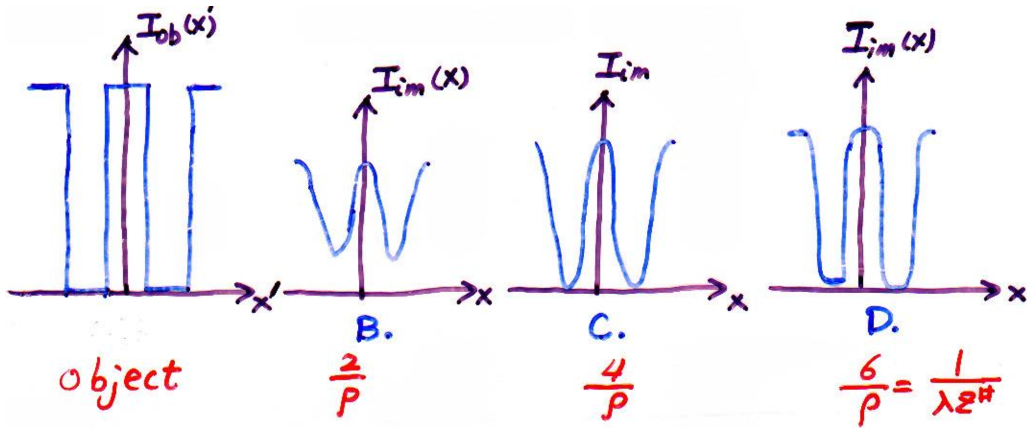
C. $\dfrac{4}{\rho} = \dfrac{1}{\lambda z^{\#}}$

D. $\dfrac{6}{\rho} \approx \dfrac{1}{\lambda z^{\#}}$



(a) (b) (c) (d)

The imaging properties were summarized in the following figure:



6.3 Image Formation with Coherent Light

Consider the case that two fields $U_1(x,t)$ and $U_2(x,t)$ are superposed on the space-time point (*x*, *t*): $U(x,t) = U_1(x,t) + U_2(x,t)$.

The total intensity becomes $I(x) = \langle U(x,t)U^*(x,t) \rangle = I_1(x) + I_2(x) + 2\,\mathrm{Re}\,\Gamma_{12}(x)$, where U satisfies the wave equation for a quasi-monochromatic light with

$$U(x,t) = A(x)e^{j\phi(x)} \cdot e^{-j2\pi\bar{\nu}t} = \tilde{A}(x) \cdot e^{-j2\pi\bar{\nu}t}.$$

6.3.1 The Imaging Problem with Coherent Light

Note $U_{im}(x,t)=\int_{-\infty}^{\infty}U_{ob}(x',t')K(x-x')\,dx'$, where $K(x-x')$ is the complex amplitude in the image plane due to a point object, *i.e.*, an amplitude impulse response of the optical system. The light intensity distribution $I_{im}(x)$ can be formed on the image plane

$$I_{im}(x)=<U_{im}(x,t)U_{im}^{*}(x,t)>=\iint <U_{ob}(x',t')U_{ob}^{*}(x'',t')>_{t'}K(x-x')K^{*}(x-x'')dx'dx''\ .$$

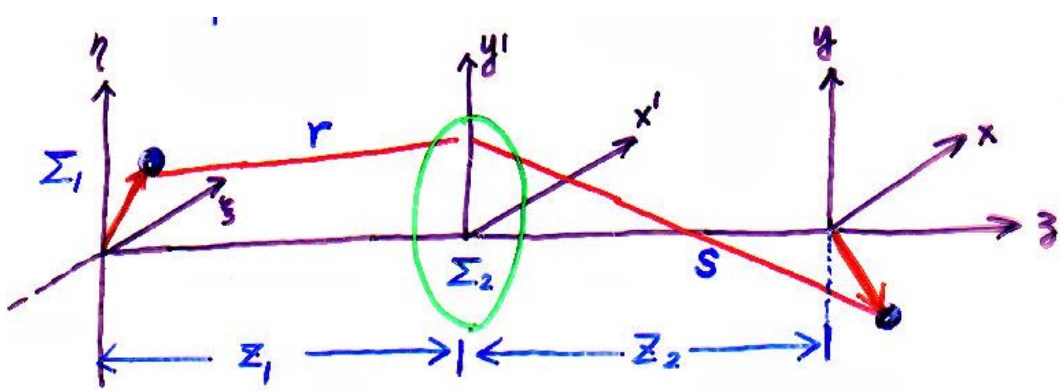
■ for an incoherent source $<U_{ob}(x',t')U_{ob}^{*}(x'',t')>_{t'}=I_{ob}(x')\delta(x'-x'')$,

$$\begin{aligned}I_{im}(x)&=<U_{im}(x,t)U_{im}^{*}(x,t)>_{t}=\int I_{ob}(x')K(x-x')K^{*}(x-x')dx'\\&=\int_{-\infty}^{\infty}I_{ob}(x')\left|K(x-x')\right|^{2}dx'=\int_{-\infty}^{\infty}I_{ob}(x')s(x-x')dx'\end{aligned}\ .$$

■ for a coherent source,

$$\begin{aligned}I_{im}(x)&=\iint U_{ob}(x')K(x-x')U_{ob}^{*}(x'')K^{*}(x-x'')dx'dx''\\&=\left|\int U_{ob}(x')K(x-x')dx'\right|^{2}\end{aligned}\ .$$

6.3.2 Amplitude Impulse Response



Referring to the optical imaging process shown above,

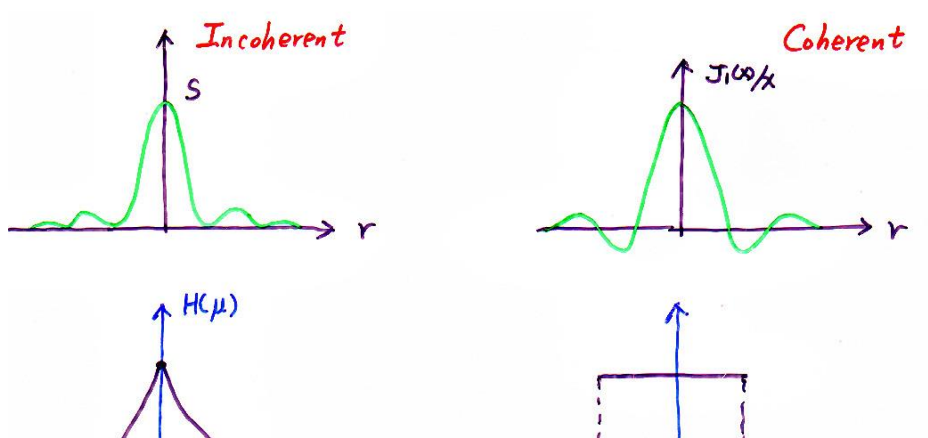
$$
\begin{aligned}
U_{im}(x,y) &= K\int_{\Sigma_2} d\,\vec{x}\,'\frac{e^{jks}}{s}\int_{\Sigma_1} d\,\vec{\xi}\,U_{ob}(\vec{\xi})\frac{e^{jkr}}{r}\cdot A(\vec{x}\,')e^{-jk\frac{|\vec{x}\,'|^2}{2f}} \\
&\overset{\substack{paraxial\ approxi. \\ and\ \frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f}}}{=} K\int_{\Sigma_2} d\,\vec{x}\,'\int_{\Sigma_1} d\,\vec{\xi}\,U_{ob}(\vec{\xi})A(\vec{x}\,')e^{-jkx'(\frac{\xi}{z_1}+\frac{x}{z_2})-jky'(\frac{\eta}{z_1}+\frac{y}{z_2})}
\end{aligned}
.
$$

Let $U_{ob}(\vec{\xi}) = \delta(\xi,\eta)$, therefore

$$
U_{im}(x,y) = K\int_{\Sigma_2} dx'dy'A(x',y')e^{-j2\pi[x'(\frac{x}{\lambda z_2})+y'(\frac{y}{\lambda z_2})]} .
$$

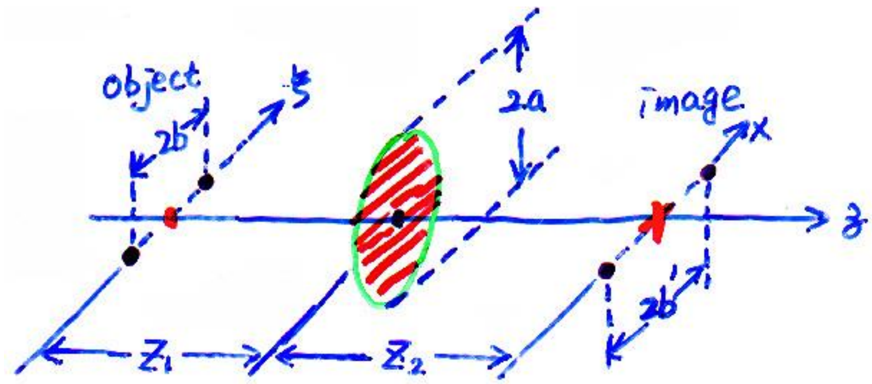
In terms of polar coordinates,

$$
\begin{aligned}
U_{im}(r,\phi) &= K\int_0^a \rho d\rho \int_0^{2\pi} e^{-j2\pi(\frac{r}{\lambda z})\rho\cos(\theta-\phi)}d\theta \\
&= 2\pi K\int_0^a J_0(\frac{2\pi r\rho}{\lambda z})\rho d\rho = K\pi a^2\,\frac{2J_1(\frac{2\pi ra}{\lambda z})}{\frac{2\pi ra}{\lambda z}} .
\end{aligned}
$$



6.3.3 Coherent Imaging Resolution

Consider an imaging process of two point sources $U_{ob}(\xi) = U_o(\xi)[\delta(\xi - b) + \delta(\xi + b)]$ in an optical system



$$\begin{aligned}\Gamma_{ob}(\xi_1,\xi_2) &= \langle U_{ob}(\xi_1)U_{ob}^*(\xi_2)\rangle \\ &= \langle U_o(\xi_1)U_o^*(\xi_2)\rangle[\delta(\xi_1 - b) + \delta(\xi_1 + b)][\delta(\xi_2 - b) + \delta(\xi_2 + b)] \\ &= I_0\,\gamma(\xi_1,\xi_2)[\delta(\xi_1 - b) + \delta(\xi_1 + b)][\delta(\xi_2 - b) + \delta(\xi_2 + b)]\end{aligned}.$$

Then $\Gamma_{im}(x_1,x_2) = \iint \Gamma_{ob}(\xi_1,\xi_2)K\left(\frac{x_1}{z_2} + \frac{\xi_1}{z_1}\right)K^*\left(\frac{x_2}{z_2} + \frac{\xi_2}{z_1}\right)d\vec{\xi}_1 d\vec{\xi}_2$, and

$$I_{im}(x) = \Gamma_{im}(x_1 = x_2) = I_0\left\{\left|K(x+b')\right|^2 + \left|K(x-b')\right|^2 + 2\,\mathrm{Re}\left[\gamma(b,-b)K(x+b')K^*(x-b')\right]\right\},$$

where $|b'| = |b|\cdot\frac{z_2}{z_1} = |b|m$.

■ 1-D system

$$K(x) = 2a\cdot sinc\left(\frac{kax}{z_2}\right).$$

$$I_{im}(x) = 4a^2 I_0\left\{sinc^2\left[\frac{ka(x-b')}{z_2}\right] + sinc^2\left[\frac{ka(x+b)}{z_2}\right] + 2\,\mathrm{Re}\left[\gamma(b,-b)sinc\frac{ka(x-b')}{z_2}sinc\frac{ka(x+b')}{z_2}\right]\right\}.$$

(a) **Incoherent Limit** with $\gamma(b,-b) = 0 \quad\Rightarrow$

$$I_{im}(x) = 4a^2 I_0 \left\{ sinc^2[\frac{ka(x-b')}{z_2}] + sinc^2[\frac{ka(x+b')}{z_2}] \right\}.$$

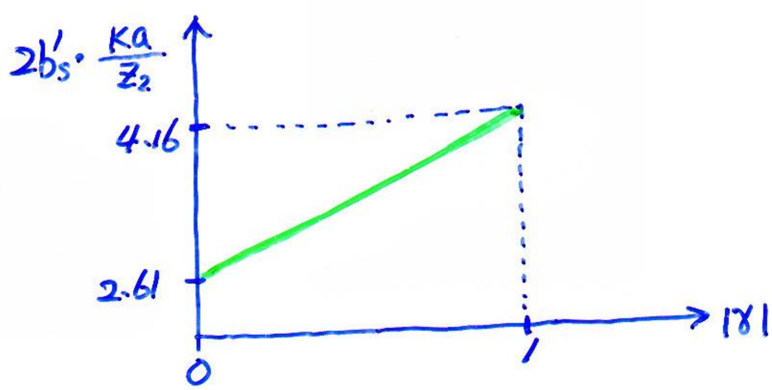
$$\text{Rayleigh Criterion}: \frac{ka(x-b')}{z_2} = 0, \quad \frac{ka(x+b')}{z_2} = \pi \quad \Rightarrow \quad 2b_r' = \pi \cdot \frac{z_2}{ka}.$$

$$\text{Sparrow Criterion}: \left.\frac{\partial^2 I_{im}(x)}{\partial x^2}\right|_{x=0} = 0 \quad \Rightarrow \quad 2b_s' = 2.606 \cdot \frac{z_2}{ka}.$$

(b) **Coherent Limit** with $\gamma(b,-b) = 1 \quad \Rightarrow$

$$I_{im}(x) = 4a^2 I_0 \left\{ sinc^2[\frac{ka(x-b')}{z_2}] + sinc^2[\frac{ka(x+b')}{z_2}] + 2\,sinc[\frac{ka(x-b')}{z_2}]sinc[\frac{ka(x+b')}{z_2}] \right\}.$$

$$\text{Sparrow Criterion}: \left.\frac{\partial^2 I_{im}(x)}{\partial x^2}\right|_{x=0} = 0 \quad \Rightarrow \quad 2b_s' = 4.164 \cdot \frac{z_2}{ka}.$$



■ 2-D system

$$K(\vec{x}) = \pi a^2 \cdot \frac{2J_1\left(\frac{ka|\vec{r}|}{z_2}\right)}{\frac{ka|\vec{r}|}{z_2}} = \pi a^2 \cdot \Lambda_1\left(\frac{ka|\vec{r}|}{z_2}\right).$$

$$I_{im}(\vec{r}) = \pi a^4 I_0 \left\{ \Lambda_1^{\,2}[\frac{ka(|\vec{r}|-b')}{z_2}] + \Lambda_1^{\,2}[\frac{ka(|\vec{r}|+b')}{z_2}] + 2\,\mathrm{Re}\left[\gamma(b,-b)\Lambda_1[\frac{ka(|\vec{r}|-b')}{z_2}]\Lambda_1[\frac{ka(|\vec{r}|+b')}{z_2}] \right] \right\}.$$

(a) Incoherent Limit with $\gamma(b,-b)=0$

Rayleigh Criterion : $2b_r' = 3.832 \cdot \frac{z_2}{ka}$.

Sparrow Criterion : $2b_s' = 2.976 \cdot \frac{z_2}{ka}$.

(b) Coherent Limit with $\gamma(b,-b)=1$

Sparrow Criterion : $2b_s' = 4.6 \cdot \frac{z_2}{ka}$.

6.4 Coherent Imaging: An Example

Consider an edge object such that

$$I_{ob}(\xi) = \begin{cases} I_0 & \xi \geq 0 \\ 0 & \xi < 0 \end{cases}.$$

■ Incoherent Illumination of the Edge

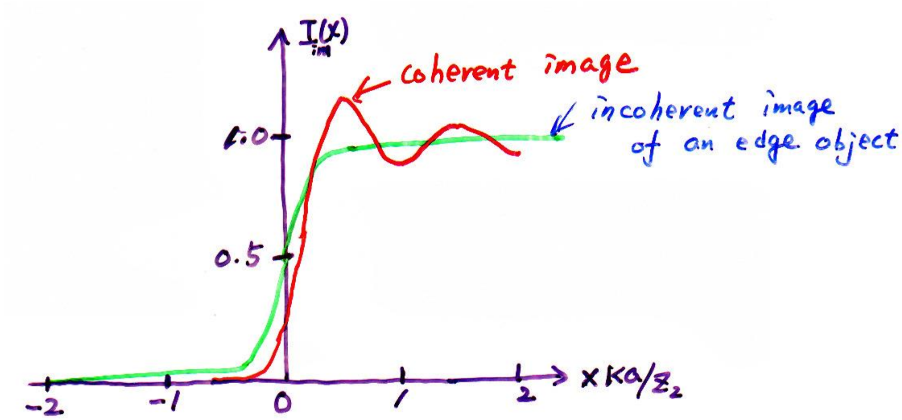
$$I_{im}(x) = \int_{-\infty}^{\infty} I_{ob}(\xi) K(\frac{x}{z_2}+\frac{\xi}{z_1}) K^*(\frac{x}{z_2}+\frac{\xi}{z_1}) d\xi$$

$$= I_0 \int_0^{\infty} sinc^2[ka(\frac{x}{z_2}+\frac{\xi}{z_1})] d\xi = \frac{I_0}{2} + \frac{I_0}{\pi} \left\{ Si(\frac{kax}{z_2}) - \frac{1-\cos(\frac{kax}{z_2})}{\left(\frac{kax}{z_2}\right)} \right\}.$$

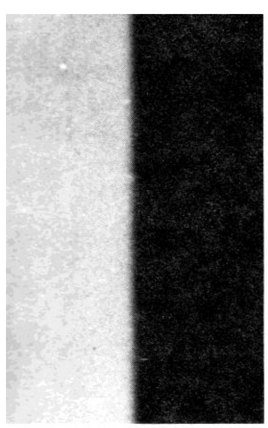
■ Coherent Illumination of the Edge with $\gamma(\xi_1,\xi_2)=1$

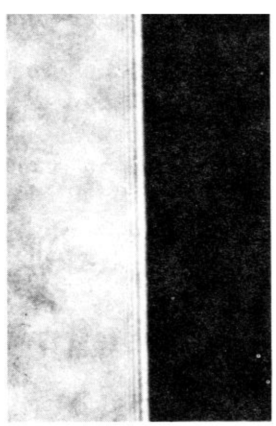
$$I_{im}(x) = \int\int \Gamma_{ob}(\xi_1,\xi_2) K(\frac{x}{z_2}+\frac{\xi}{z_1}) K^*(\frac{x}{z_2}+\frac{\xi}{z_1}) d\xi_1 d\xi_2 = \left| \int \sqrt{I_{ob}(\xi)} K(\frac{x}{z_2}+\frac{\xi}{z_1}) d\xi \right|^2$$
$$= I_0^{\,2} \left[\frac{1}{2} - \frac{1}{\pi} Si(\frac{kax}{z_2}) \right]^2 ,$$

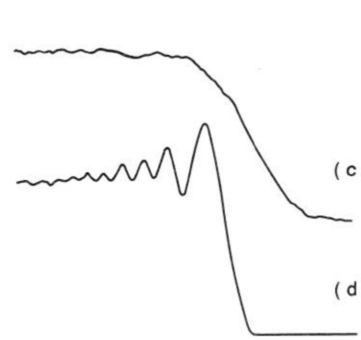
where $Si(x) = \int_0^x \frac{\sin u}{u} du$. For your reference, the image distribution $I_{im}(x)$ as a function of $x \cdot ka/z_2$ is shown in the following:



The photograph showing the edge patterns with incoherent (a) and coherent imaging (b) are shown below. The corresponding traces across the edge are presented on (c) and (d).







(a) (b) (c) and (d)