# Statistical Optics and Image Formation

**Homework Set 1** **Due: Oct. 28, 2020**

1. **Multiple-Scattering Model of Turbulence (40%)**
The irradiation at a pixel in the image of a star fluctuates randomly due to atmosphere turbulence. Provided the exposure time is brief enough, we can model the turbulent medium as the parallel slabs, each of which scatters away a random amount of an incident light amplitude. If $t_n$ denotes the amplitude transmittance of the nth slab, the amplitude $u$ at the imager, which lies within the scattering medium, obeys a simple product $u = u_0 \cdot \prod_{i=1}^{N} t_i$ . Here $u_0 = 1$ is the initial amplitude and $N$ is the total number of slabs.
   (a) By taking the natural logarithm of $u = \prod_{i=1}^{N} t_i$ , find the conditions on the $t_i$ and $N$ for which $\ln|u|$ is a normal *RV*.
   (b) The irradiance $I = |u|^2$ . Show that $I$ is a log normal *RV*.
   (c) Attached file **irad.mat** is a data set of 400 observations on $I$. Find the associated probability density law. Give the mean and variance of normal distribution of $\ln(|u|)$ .

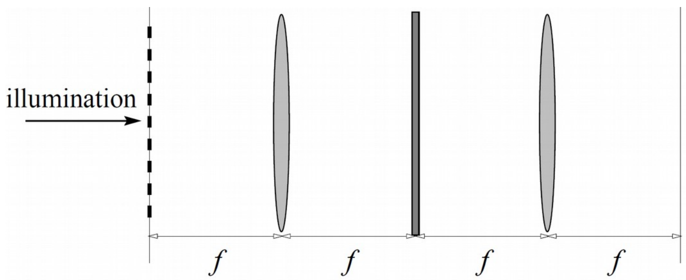
2. **Sum of n Random Variables** (20%)
Let $M$n denote the sequence of sample means from an independent and identical distribution (iid) random process $X_n$ : $X_n = (X_1 + X_2 ... + X_n)/n$ .
(a) Is $M$n a Markov process?
(b) If the answer of part (a) is yes, find the conditional probability density function (pdf) $P_{M_n}(x \mid M_{n-1} = y)$ , i.e., the state transition pdf of the Markov process. If no, find the corresponding pdf of $M_n$ , $P_{M_n}(x)$ .

3. **<u>Optical Signal Detection</u>** (40%)
The number of electrons counted $X$ by a photon detector obeys Poisson statistics and is therefore a Poisson random variable specified by an average arriving rate $\lambda$. In matlab, you can generate such a signal string using y=poissrnd( $\lambda$ , 1, N), where $N$ is the number of data points. The electronics of your photon detector also produces a thermal noise $N(0,\sigma^2)$. Therefore, the total signal from the photon detector can be simulated with a generative model of $y = poissrnd(\lambda,1,N) + \sigma * randn(1,N)$ . The attached file "**pstream.mat**" contains a 10-s lomg stream signal with unknown $\lambda$ Poisson statistics. Each counting interval is 10 ms, yielding $N$=1000. Assume the photon generating process and thermal noise to be statistically independent and $\sigma = 3$ for the Gaussian-distributed thermal noise.
(a) Invoke Bayes theorem and use a grid search technique to estimate the posterior probability with a uniform prior pdf. Plot the posterior pdf on the grid of model parameter $\lambda$ .

(b) Use a gradient-based least-square error minimization technique to find out the optimal value of $\lambda$. What is the best estimate value of the photon arriving rate $\lambda$? Does your best estimate agree with the model parameter at the peak of posterior pdf shown in (a)?

**4. <u>Imaging process with white noise</u> (20%)**

An object, which exhibits a periodic square-wave amplitude transmittance (period= $X$), is imaged by a $4f$ imaging system. The focal length of the lens is $f$=10 cm.




The fundamental frequency of the square wave is $1/X$=100 cycles/mm, the object is placed at $z$=$f$=10 cm, and the wavelength of light source used is 0.6 μm. Assume the imaging system to have a circular pupil function. What is the minimum lens diameter that can yield any variations of intensity across the image plane for the cases of

(a) a coherent object illumination with optical field?

(b) an incoherent object illumination with optical irradiance?

(c) If a bright background appears in the incoherent illumination, design an annular pupil (with appropriate $R_1$ and $R_2$) to remove the background while still yields detailed variations of intensity across the image plane.

**Note:** The Fourier series of the square wave can be expressed as follows

$$t(x) = \frac{1}{2} \sum_{n=-\infty}^{n=+\infty} sinc\left(\frac{n}{2}\right) exp\left\{j2\pi n \frac{x}{X}\right\}$$