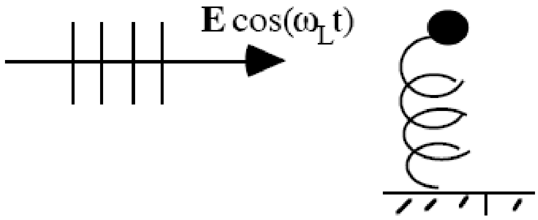
Homework 1 of Nonlinear Optics (Due: December 3, 2007)

1. (30 points) **Lorentz Classical Model of Absorption and Emission**

Suppose we were to model an atom as an electron on a spring - *i.e.* a damped simple harmonic oscillator of mass *m*, with resonance frequency ω_0, and damping constant Γ.



(a) Show that rate at which the dipole absorbs energy from the field, given by the rate at which the field does work on the charge averaged over one period, is

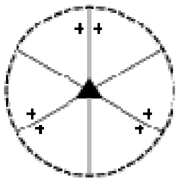
$$\frac{dW_{abs}}{dt} = \frac{\pi e^2 |E|^2}{4m} g(\omega_L), \quad where\ g(\omega) = \frac{\Gamma/(2\pi)}{(\omega - \omega_{eg})^2 + (\Gamma^2/4)}$$

Assume near resonance so that $\Delta = \omega_L - \omega_0 \ll \omega_L, \omega_0$.

(b) The absorption cross section, σ_{abs}, is defined as the rate at which energy is absorbed by an atom, divided by the flux, $\Phi = I/\hbar\omega_L$, (*i.e.* the rate of photons incident on the atom per unit area), where $I = \frac{c}{8\pi}|E_0|^2$ is the incident intensity (CGS units). Show that the classical model of absorption gives,

$\sigma_{clasical} = \frac{2\pi^2 e^2}{mc} g(\omega_L)$. Evaluate this on-resonance, for the parameters associated with ***Na***, where the excitation wavelength is 589 nm and the line width (Full width at half-maximum) is 10 MHz.

2. (30 points) The form of the second-order susceptibility tensor for the crystal $LiNbO_3$ belonging to 3m (C_{3v}) can be determined by Neumann's principle as shown in the lecture. The point-symmetry group of 3*m* indicates that crystal has several classes of symmetry operations {E, $2C_3$, $3\sigma_v$}, which include identity operation E, three-fold rotations $2C_3$, and three vertical mirror planes $3\sigma_v$, as described below



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Using these symmetry operations to deduce all nonvanishing components and determine how many independent components in this susceptibility tensor? By invoking Kleimann symmetry, what is the final form of $\chi^{(2)}$.

3. (20 points) Following to the previous question, we can find that type-I ($o_{\lambda} + o_{\lambda} \rightarrow e_{\lambda/2}$) phase matching can be achieved with $LiNbO_3$ at $\lambda = 1.06$ μm. Calculate the resulting effective nonlinearity.

4. (20 points) Show why overall permutation symmetry and time-reversal invariance breaks down when any of the optical frequencies nears a transition frequency of an optical medium.