Homework Set 4
Linear Algebra and Vector Calculus

Due: June 20, 2008

1. .A vector field satisfies $\nabla \times \vec{v}(\vec{r}) = 0$ everywhere, then it follows that you can write $\vec{v}(\vec{r})$ as the gradient of a scalar function, $\vec{v}(\vec{r}) = -\nabla \phi(\vec{r})$. For each of the following vector fields find a function $\phi(\vec{r})$ that does this. First determine is the curl is zero, because if it isn't then your hunt for a will be futile.
$\vec{v}(\vec{r}) = \hat{x} y \cos(xy) + \hat{y} x \cos(xy)$

2. What is the total flux, $\oint \vec{E}(\vec{r}) \cdot d\vec{A}$ with $\vec{E}(\vec{r}) = \alpha x \hat{x} + \beta y \hat{y} + \gamma z \hat{z}$, out of the cube of side a with one corner at the origin? .

3. Compute $\nabla \phi(\vec{r})$ with $\phi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$. Also compute the Laplacian of the same exponential, $\nabla^2 \phi = div\ grad \phi$.

4. By analogy to $\nabla \cdot \vec{v} = \lim_{V \to 0} \frac{1}{V} \frac{dV}{dt} = \lim_{V \to 0} \frac{1}{V} \oint \vec{v} \cdot d\vec{A}$ and $\nabla \times \vec{v} = \lim_{V \to 0} \frac{1}{V} \oint d\vec{A} \times \vec{v}$ verify $\lim_{V \to 0} \frac{1}{V} \oint \phi(\vec{r}) d\vec{A} = \nabla \phi(\vec{r})$ by computing this in rectangular coordinates and showing that it has the correct components. In the same spirit as the derivation of Gauss's theorem, verify the identity $\oint_S \phi d\vec{A} = \int_V \nabla \phi dV$.

5. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ for $\vec{F}(\vec{r}) = Axy\hat{x} + Bx\hat{y}$ around the circle of radius R centered at the origin. Now do it again, using Stokes' theorem this time.